Course Development and Active Learning

A retrospective spanning nine courses

University of Wisconsin-Madison

1993: Math 130/131. Arithmetic/Geometry for teachers
1994: Math 132. Quantitative reasoning for teachers
1995: Math 219. Mathology: a freshman seminar breadth course about math

Ohio State University (very similar courses)

2001: Math 105-106. Arithmetic/Geometry for teachers
2003: Math 107. Quantitative reasoning for teachers

University of Pennsylvania

2014: Math 110. Calc "1.5" for business majors who know calculus but not very well
2018: Math 546. Applied probability modeling with measure theory foundation
2021: Math 107/108. Single and multivariable calculus for the inexact sciences
2025: Math 025. Geometry and art (freshman seminar for nonmajors)

SOME PRE-HISTORY

This is my fiftieth year in the Math Education business

- 1976 precalculus “my way”
- 1979-1984 math enrichment for grades 5-8
- 1988-1989 early summer workshops in Treisman’s Emerging Scholars program (Mills College 1988, UC Berkeley 1999)

Training with ex-Project SEED personnel 1979-1981

A life-changing experience – long story – you can ask me later

Why active learning?

Why active learning?

NOT THE ANSWER: Because it is Evidence-Based. 👎🏽👎🏽

1. In my opinion, the evidence is weak and exhibits confirmation bias. Very few controlled experiments (understandably!).
2. What confirming data there is appears much stronger in science than in math.
3. Most importantly: dependent variables appear to be those associated with traditional pedagogies.

Active learning is for accomplishing goals best accomplished in an active learning environment!

Why test it on goals for which traditional lectures have been optimized?

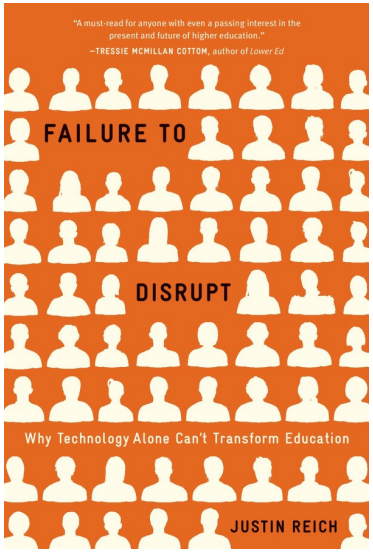
One would not expect active learning to do particularly better on those, though it's my suspicion that a steady diet of active learning for 12 years will in fact beat traditional methods on their own goals.

How would you teach your own children?

Pause for a
1-minute video

REFORM OR REVOLUTION

To tweak or to rebuild from scratch?

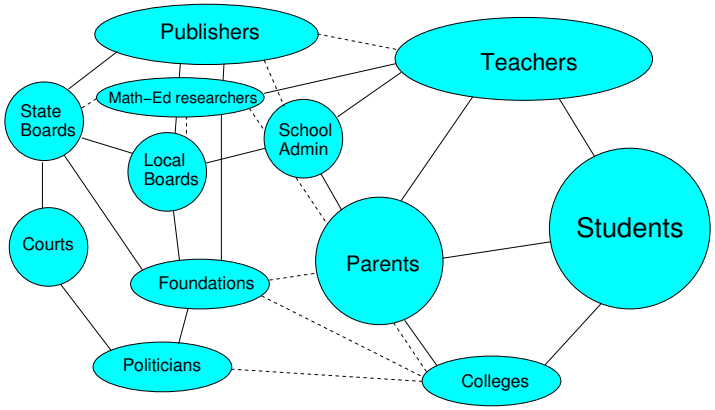


Quoting Justin Reich:

"... schools are conservative institutions in society."

Rebuilding from scratch is a huge pain. But there are also dangers from the incremental approach.

The system is stuck in a local potential well with. HUGE restoring forces.



RESTORING FORCES

- Textbooks
- Teacher competencies
- Standardized tests
- Placement discontinuities
- What Reich calls “the curse of the familiar”

Conclusions:

1. Either approach could be right for you
2. Watch out for the hidden cost of it all being a waste of time
3. If you rebuild from scratch, figure out the goals and be led by them.

Calculus for the inexact sciences meta-curriclum

Mental skills

- Concentration
- Abstraction
- Perseverance
- Argumentation

Verbal skills

- Proper grammar and precise terms
- Articulating statements
- Listening to others
- Clear enough to teach

Attitudes

- Ownership
- Curiosity
- Skepticism
- Taste for argumentation

Logical and syntactic skills

- Free and bound variables
- Quantifiers (yikes!)
- Propositional logic
- Recognizing counterexamples

Modeling skills

- Naming and assigning constants, variables and functions
- Inverse functions
- Units of all the above
- Units of integrals and derivatives

Problem solving skills: the usual handful from, e.g., Pólya

WHY THESE SOFT SKILLS DICTATE AN ACTIVE PEDAGOGY

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Address this question interactively

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Who is this meta-curriculum for?

- Students who don't already have these skills
- The older they are, the less they should need to be taught how to learn, but this is not always the case
- Those who need to use the mathematics for modeling, for understanding big-picture relationships and for obtaining intuition about complex systems => students in the inexact sciences

- Not for students who need precision, speed and a library of sophisticated algorithms and knowledge of when to use them

Active learning at the graduate level?

Math 5460: Applied probability modeling for Masters level mathematics students and PhD level students in math-adjacent fields

Requirements: Undergraduate real analysis, some exposure to probability

These students need to understand the important constructions and statements of theorems. They will concentrate more on using these than proving them, except in the (many) instances where the proofs shed light on meaning and intuition.

Contrast to the course for Math and Stat PhD students. They already have some idea of how to connect formal math to intuition and physical models. For those students, what is needed is to absorb the classical constructions and theorems and their proofs. They need proofs with commentary, a guide through the hefty apparatus of measure-theoretic probability, annotated with examples and applications. They need to be able to interrogate the lecturer who is giving the proofs, and to absorb the proofs well enough to be able to prove new things from the toolkit of known methods.

HOW TO PULL THIS OFF

- In-class worksheets
  - Keep updating – trial and error
  - Of course, low-threshold, high-ceiling
  - Flexible design so you don't have to commit to completing
  - More scaffolding than you first imagine will be needed
  - Don't pack too much in
  - Don't expect to be done tweaking: every new cohort is a bit different

Institutionalization:

Most students find active learning sessions leave them unsure of what turned out to be right and to be a good way of attacking and encapsulating the problem. Save time for a wrap-up that lets them know these things.

Post follow up information immediately: nicely modeled solutions, a sheet of definitions, etc.

Devote time to going over problems from homework as well as those done in class. Many students can't, in real time, take in the beautiful lesson you led them to.

BIG PICTURE

→ I only develop a new active learning class or sequence once every ten years or so. It's time consuming!

Think ahead about personnel. It won't work if you're the only one to teach it. It's hard to develop a cadre of other teachers to teach the course.

It requires repetition to get it right. More than that, other sets of eyes are crucial to improving the course and fixing anything that isn't working. Inter-visits make everything twice as good!

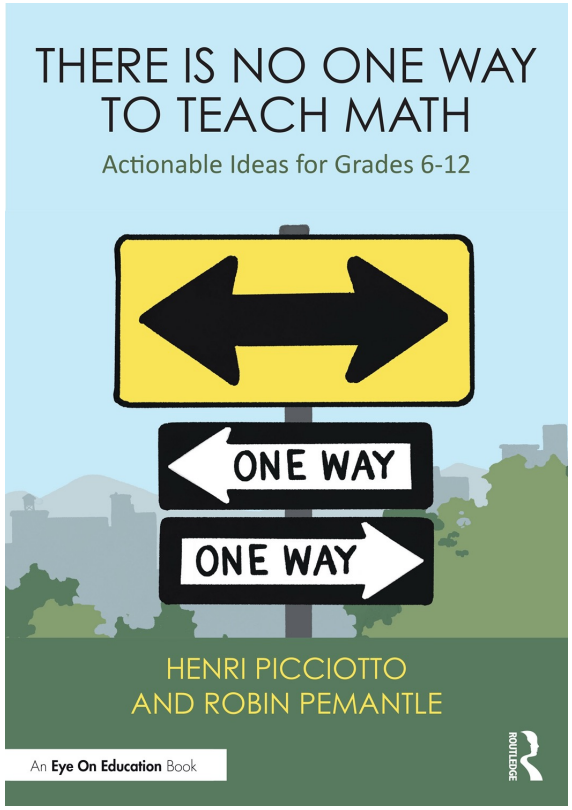
If you don't have colleagues to do this with, you can try your school's teaching and learning center. If you don't have that either, consider not taking on this adventure.

What is novel in higher ed is sometimes well known in K-12 circles.

Engage someone who teaches kids to help you brainstorm!

Doing this, I learned about how best to lag homework, how to plant ideas informally that will be taught in depth later, how to use formative evaluations and "test corrections" to double the amount of learning, how to insert new learning into skills drills, and many other K-12 devices.

All this advice and more can be found in my recent book with Henri Picciotto, THERE IS NO ONE WAY TO TEACH MATH:



I will also take this opportunity to plug my forthcoming Undergraduate Math Topics book with Julian Gould: