



# **Building Metacognitive Skills in Calculus (& Beyond)**

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# Outline

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- What is metacognition & why is it important?
- How can we build metacognition into our courses?
- An example: learning strategies in calculus
- Reflecting & adapting to proof-based courses



# Learning to Learn

*“Metacognition is awareness and control of thinking for learning.”*

(Cross, Paris ‘88)

*Wisdom is not a product of schooling  
but of the life-long attempt to acquire it.*

– Albert Einstein



# Metacognition

```
graph TD; Metacognition --> Metacognitive_Knowledge; Metacognition --> Metacognitive_Regulation; Metacognitive_Knowledge --> Declarative_knowledge; Metacognitive_Knowledge --> Procedural_knowledge; Metacognitive_Knowledge --> Conditional_knowledge; Metacognitive_Regulation --> Planning; Metacognitive_Regulation --> Monitoring; Metacognitive_Regulation --> Evaluating;
```

## Metacognitive Knowledge

Declarative knowledge

Procedural knowledge

Conditional knowledge

## Metacognitive Regulation

Planning

Monitoring

Evaluating

Knowing yourself as a learner and which cognitive procedures are best suited for learning  
e.g. knowing that learning is boosted by practice as opposed to simply rereading course content

# Metacognition

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```

## Metacognitive Knowledge

## Metacognitive Regulation

Declarative knowledge

Procedural knowledge

Conditional knowledge

Planning

Monitoring

Evaluating

Knowing how to perform a cognitive-based procedure

e.g. Knowing that practicing solving problems before viewing solutions boosts performance

# Metacognition

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```

## Metacognitive Knowledge

## Metacognitive Regulation

Declarative knowledge

Procedural knowledge

Conditional knowledge

Planning

Monitoring

Evaluating

Knowing when and why to use a cognitive-based procedure

e.g. knowing that multiple short practice sessions as opposed to one long practice session boosts performance

# Metacognition

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## Metacognitive Knowledge

## Metacognitive Regulation

Declarative knowledge

Procedural knowledge

Conditional knowledge

Planning

Monitoring

Evaluating

e.g. Developing a study plan before an exam with concrete goals and timeline

# Metacognition

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```

## Metacognitive Knowledge

## Metacognitive Regulation

Declarative knowledge

Procedural knowledge

Conditional knowledge

Planning

Monitoring

Evaluating

e.g. Regularly checking in on your progress while working through your study plan

# Metacognition

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graph TD; Metacognition --> Metacognitive_Knowledge; Metacognition --> Metacognitive_Regulation; Metacognitive_Knowledge --> Declarative_knowledge; Metacognitive_Knowledge --> Procedural_knowledge; Metacognitive_Knowledge --> Conditional_knowledge; Metacognitive_Regulation --> Planning; Metacognitive_Regulation --> Monitoring; Metacognitive_Regulation --> Evaluating; style Evaluating fill:#add8e6
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## Metacognitive Knowledge

Declarative knowledge

Procedural knowledge

Conditional knowledge

## Metacognitive Regulation

Planning

Monitoring

Evaluating

e.g. Reflecting on the success of your study plan after the exam

# Think – Pair – Share:

## When studying for an exam...

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### What do students do?

- Use passive strategies in which they reread or memorize class materials<sup>1</sup>
- Procrastinate and cram studying into one large session
- Focus on how to respond to question format and wording
- Proceed without a plan<sup>2</sup>
- Avoid asking for support and advice

### What should students do?

- Actively practice, self-test, and obtain feedback on understanding
- Space out studying over multiple short sessions
- Focus on concepts and problem solving
- Plan how to manage time, what strategies to use, and what to focus on
- Seek support from trusted mentors and adapt study strategies

1. Stanton, Sebesta, Dunlosky. *Evidence Based Teaching Guide: Student Metacognition*. 2021

2. Halmo, Yamini, Stanton. *Metacognition and Self-Efficacy in Action: How First-Year Students...* 2024

# Why metacognition?



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- Meta-analyses of Hattie (2009) and Donker et al., (2014) find teaching approaches that emphasize student metacognitive skills are **among the most effective teaching approaches**.
- Strong metacognitive skills are linked with increased academic performance (Ohtani & Hisasaka '18, De Boer et al. '18, Guo '22) **problem solving skills, and math performance** (Hacker et al. '98, Mevarech & Amrany, '08, Jacobse & Harskamp, '12, Desoete & De Craene, '19, Muncer et al. '22).
- “Controlling teaching” results in avoidance of studying (Hein et al. '15) whereas support for student autonomy and **metacognitive self-regulation increased student commitment** (Yu et al. '16).
- Strong metacognitive skills are linked with **improved sense of self-efficacy** and academic achievement (Ambrose, et al. '10, Pintrich & De Groot, '90, Pajares, '96, Pajares, '02; Huang et al., '22).

# What can we do to incorporate metacognition in our courses?

- Encourage explicit planning, monitoring, and evaluating when they study<sup>1</sup>
- Encourage students to use **self-testing**, **spacing**, and **interleaving** when they study
  - **active practice**, multiple short sessions, **mixing topics**
- Provide materials to support self-testing and access to feedback
- Develop clear learning goals
- Incorporate opportunities to reflect on learning
  - e.g. muddiest point, assessment wrappers
- Incorporate **social metacognition** into groupwork through **reciprocal scaffolding** and **collective strategizing**
  - **Repeat, elaborate, and question one another's ideas & collaborate to develop, assess, and modify approach**

# Example: Learning Strategies in Calculus

## Calculus of Functions of One Variable II

Math 115, Fall 2022

### Weekly Learning Strategies

In Math 115, we want to help you build effective learning strategies that you can use throughout your time at Yale and beyond. Each week, we will ask you to try out and briefly reflect on a new learning strategy. **Learning Strategy reflections are posted on the Canvas umbrella site are due on Thursdays at 11:59pm.** You will earn points toward your exploration grade for completing each reflection.

Week 1	Build a Network
Week 2	Actively Engage
Week 3-4	Plan & Prepare
Week 5	Exam 1
Week 6	Make Adjustments
Week 7	Review & Practice
Week 8	
Week 9-10	Plan & Prepare
Week 11	Exam 2
Week 12	Make Adjustments
Week 13	
Week 14	Reflect & Grow

# The Weekly Schedule

Week 1	Build a Network
Week 2	Actively Engage
Week 3-4	Plan & Prepare
Week 5	Exam 1
Week 6	Make Adjustments
Week 7	Review & Practice
Week 8	
Week 9-10	Plan & Prepare
Week 11	Exam 2
Week 12	Make Adjustments
Week 13	
Week 14	Reflect & Grow

## Goal: Build a Network

We find that successful Math 115 students build a network of peers that they can work and study with throughout the semester. Here are some strategies you could try:

### Strategy 1: Attend peer tutoring

Why not give it a try, and drop in sometime next week? Our Math 115 peer tutors are excellent resources -- watch the video below to learn more about the peer tutor program.

### Strategy 2: Form a study group

Why not give it a try? To get started, take some time to get to know your classmates, exchange contact information, and set up a time to work on the first problem set together.

You will be asked to reflect on how these strategies worked for you in next week's Explore.

Week 1	Build a Network
Week 2	Actively Engage
Week 3-4	Plan & Prepare
Week 5	Exam 1
Week 6	Make Adjustments
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Week 13	
Week 14	Reflect & Grow

## Reflection: Building a Network

Last week's goal was to begin **building a network** by **attending peer tutoring** and **forming a study group**.

Take a few minutes to reflect on how this learning strategy is working for you so far. There are no wrong answers here! The point is to try new strategies, reflect, and (hopefully) discover something about how you learn best and ways you can improve as a learner.

### Question 1

5 pts

In a few sentences, reflect on your experience trying to build a network. Do you think that attending peer tutoring and/or working in a study group are helpful learning strategies to build into your weekly routine? Did you learn anything about the way you learn?

Week 1	Build a Network
Week 2	Actively Engage
Week 3	Actively Engage
Week 4	Actively Engage
Week 5	Actively Engage
Week 6	Actively Engage
Week 7	Actively Engage
Week 8	Actively Engage
Week 9-10	Plan & Prepare
Week 11	Exam 2
Week 12	Make Adjustments
Week 13	Make Adjustments
Week 14	Reflect & Grow

## Reflection: Building a Network

*“Interacting with a [group] eases my anxiety about asking questions. [Often], we share the same questions which gives us an opportunity to brainstorm and learn from each other. Sometimes when someone elaborates on their thinking it aligns with mine so much that it allows me to understand how to move forward.”*

*“Practicing verbally explaining concepts and solutions to others was surprisingly useful for my learning because it required me to first consolidate what I knew.”*

*“I learned that [working in a group] is most effective when I try to look over the problem sets beforehand, so I know what I struggle with.”*

<b>Question 1</b>	<b>5 pts</b>
<p>In a few sentences, reflect on your experience trying to build a network. Do you think that attending peer tutoring and/or working in a study group are helpful learning strategies to build into your weekly routine? Did you learn anything about the way you learn?</p>	
<div style="border: 1px solid #ccc; height: 40px;"></div>	

Week 1	Build a Network
Week 2	Actively Engage
Week 3-4	Plan & Prepare
Week 5	Exam 1
Week 6	Make Adjustments
Week 7	Review & Practice
Week 8	
Week 9-10	Plan & Prepare
Week 11	Exam 2
Week 12	Make Adjustments
Week 13	
Week 14	Reflect & Grow

## Goal: Actively Engage

We find that successful Math 115 students actively engage with the material both inside and outside the classroom. Here are some strategies you could try:

### Strategy 1: Participate Actively in Class

Ask questions, share ideas, and don't be afraid to make mistakes! Speaking up in class and contributing during group-work can help us to get quick feedback on our understanding of a topic, and prevents us from getting lost or falling behind in class. Why not give it a try?

### Strategy 2: Attend your Instructor's Office Hours

Even if you don't have any specific questions in mind, office hours provide opportunities to practice and engage with the material in a collaborative and supportive environment. Why not give it a try?

You will be asked to reflect on how these strategies worked for you in next week's Explore.

Week 1	Build a Network
Week 2	Actively Engage
Week 3-4	Plan & Prepare
Week 5	Exam 1
Week 6	Make Adjustments
Week 7	Review & Practice
Week 8	
Week 9-10	Plan & Prepare
Week 11	Exam 2
Week 12	Make Adjustments
Week 13	
Week 14	Reflect & Grow

### Reflection: Actively Engaging

Last week's goal was to **actively engage** in the course by **participating actively in class** and **attending office hours**.

Take a few minutes to reflect on how this learning strategy is working for you so far. There are no wrong answers here! The point is to try new strategies, reflect, and (hopefully) discover something about how you learn best and ways you can improve as a learner.

<b>Question 2</b>	<b>5 pts</b>
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What could you do to improve your participation in the course and improve your learning?

Week 1	Build a Network
Week 2	Actively Engage
Week 3-4	Plan & Prepare

## Reflection: Actively Engaging

*“Participating actively in class has been really helpful to me because I am able to get **immediate feedback on my understanding**. One way to improve my learning would be to participate in class even if I don't feel confident that I have the right answer.”*

*“The biggest change I need to make [is to improve] my willingness to be wrong. Previously, if I raised my hand in class, it meant I absolutely knew the answer. However, **answering incorrectly [seems] to help me learn more effectively**, since now both the teacher and I know where my thinking leads.”*

*“I [tried to] answer some questions in class this week and made an effort to talk to those around me about the problems we were doing in class, but I [need to work on] responding to questions no matter whether I am wrong or right [because] **being scared of getting the answer wrong holds me back from learning**.”*

Week 12	Make Adjustments
Week 13	
Week 14	Reflect & Grow

What could you do to improve your participation in the course and improve your learning?

Week 1	Build a Network
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Week 3-4	Plan & Prepare
Week 5	Exam 1
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Week 7	Review & Practice
Week 8	
Week 9-10	Plan & Prepare
Week 11	Exam 2
Week 12	Make Adjustments
Week 13	
Week 14	Reflect & Grow

## Goal: Plan and Prepare

We find that successful Math 115 students begin thinking about how they will prepare for exams well in advance. They plan out how they will manage their time and adapt their plan as necessary. Here is a strategy you could try:

### Strategy: Create a Two-Week Study Plan

Our first exam is on 9/29 -- that's 2 weeks away! Now is a great time to begin deciding how you will manage your time, so why not give it a try? To begin creating your study plan, you might want to think about the following questions:

- When will I complete my problem sets?
- When will I complete my work for other classes?
- When will I attend office hours?
- Which topics do I feel confident about? Which ones am I still struggling on (I will need to spend more time on these)?

You might want to collaborate with your peers as you make your study plan, but remember: every study plan may look different -- we all have different schedules and different study needs! If you are not sure about your study plan, you may want to run it by your instructor or a peer tutor and get some advice. In case you need some inspiration, here are some [sample study plans](#) ↓ .

Week 1	Build a Network
Week 2	Actively Engage
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Week 5	Exam 1
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Week 7	Review & R
Week 8	

### Goal: Plan and Prepare

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**Strategy: Create a Two-Week Study Plan**

<b>Question 1</b>	<b>3 pts</b>
Use this space to brainstorm some ideas and goals for your study plan.	

*I plan to **space out my studying into [short daily] review sessions [consisting] of a mini lesson and then practice questions for that lesson. I hope to finish reviewing all the content 2-3 days before the test so that I can spend the last few days [doing] practice questions.***

Week 12	Make Adjustments
Week 13	
Week 14	Reflect & Grow

You might want to collaborate with your peers as you make your study plan, but remember: every study plan may look different -- we all have different schedules and different study needs! If you are not sure about your study plan, you may want to run it by your instructor or a peer tutor and get some advice. In case you need some inspiration, here are some [sample study plans](#) ↓ .

Week 1	Build a Network
Week 2	Actively Engage
Week 3-4	Plan & Prepare
Week 5	Exam 1

### Goal: Plan and Prepare

We find that successful Math 115 students begin thinking about how they will prepare for exams well in advance. They plan out how they will manage their time and adapt their plan as necessary. Here is a strategy you could try:

*“I feel like I have not been able to manage my time effectively, so I am going to try [to designate] daily goals to complete. This way, I review a little bit at a time instead of cramming all of the material within a few days.”*

*“My studying is going great so far. I am trying to take 30 [minutes] or so to study everyday before the test.”*

*“I think my studying is going pretty well as I have spent some time reviewing concepts from earlier in the course. However, I think I should dedicate more time to practical applications of what I am studying by doing some practice problems and hopefully starting the practice exams in the next few days.”*

Week 11	Exam 2
Week 12	Make Adjustments
Week 13	
Week 14	Reflect & Grow

#### Question 1

3 pts

How is your studying going so far? Do you need to make any adjustments to your time management?

Week 1	Build a Network
Week 2	Actively Engage
Week 3-4	Plan & Prepare
Week 5	Exam 1
Week 6	Make Adjustments
Week 7	Review & Practice
Week 8	
Week 9-10	Plan & Prepare
Week 11	Exam 2
Week 12	Make Adjustments
Week 13	
Week 14	Reflect & Grow

## Goal: Plan and Prepare

We find that successful Math 115 students begin thinking about how they will prepare for exams well in advance. They plan out how they will manage their time and adapt their plan as necessary. Here is a strategy you could try:

## Reflection: Planning and Preparing

Our last goal was to **plan and prepare** for the first exam by **creating a two-week study plan**.

Take a few minutes to reflect on how this learning strategy worked for you. There are no wrong answers here! The point is to try the strategy, reflect, and (hopefully) discover something about how you learn best and ways you can improve as a learner.

### Question 1

5 pts

Think about your study plan. Which parts of your plan worked well for you? Is there anything you would change for next time?

Week 1	Build a Network
Week 2	Actively Engage
Week 3-4	Plan & Prepare
Week 5	Exam 1

### Goal: Plan and Prepare

We find that successful Math 115 students begin thinking about how they will prepare for exams well in advance. They plan out how they will manage their time and adapt their plan as necessary. Here is a strategy you could try:

### Reflection: Planning and Preparing

*“Making a short study guide for each section [was] helpful for me. Next time [I plan] to **complete the concept checks after each class (rather than completing them all at the end)** so that I am regularly refreshing my understanding.”*

*“I knew all the basic concepts but [lacked] the ability to apply those concepts to more difficult problems. Next time I'd like [to dedicate] more time to practicing and applying my math instead of just going over the concepts.”*

*“Making a study plan worked very well for me as it helped me **divide my studying into manageable portions and focus on my trouble areas** without getting distracted or overwhelmed. I don't think there is much I would change for next time.”*

*“Next time, I will [start studying] **well in advance** so I am not rushing through my studying, and instead, [developing] a deeper understanding of the mathematical concepts.”*

Week 13	
Week 14	Reflect & Grow

Think about your study plan. Which parts of your plan worked well for you? Is there anything you would change for next time?

Week 1	Build a Network
Week 2	Actively Engage
Week 3-4	Plan & Prepare
Week 5	Exam 1
<b>Week 6</b>	<b>Make Adjustments</b>
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Week 8	
Week 9-10	Plan & Prepare
Week 11	Exam 2
Week 12	Make Adjustments
Week 13	
Week 14	Reflect & Grow

## Goal: Make Adjustments

We find that successful Math 115 students use the feedback they get on exams and homework to make adjustments to how they solve problems and approach the course. They think of mistakes as opportunities to learn, analyzing where they went wrong and how to correct their thought processes.

### Strategy: Work through Exam 1 and make corrections

Now that you have received feedback on the first exam, it's a good time to reflect on how it went, correct any misconceptions you have, and fix any mistakes you made. You may want to try making corrections on your own at first, then drop by peer tutoring or office hours if you run into any issues. Why not give it a try?

You will be asked to reflect on how this strategy worked for you in next week's Explore.

Week 1	Build a Network
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Week 5	Exam 1
<b>Week 6</b>	<b>Make Adjustments</b>
Week 7	Review & Practice
Week 8	
Week 9-10	Plan & Prepare
Week 11	Exam 2
Week 12	Make Adjustments
Week 13	
Week 14	Reflect & Grow

## Goal: Make Adjustments

### Reflection: Making Adjustments

Last week's goal was to begin **making adjustments** by **working through Exam 1 to make corrections.**

Take a few minutes to reflect on how this learning strategy worked for you. There are no wrong answers here! The point is to try the strategies, reflect, and (hopefully) discover something about how you learn best and ways you can improve as a learner.

#### Question 1

5 pts

In a few sentences, reflect on what you learned as you worked through the exam. Are there any topics you would like to work on to improve your understanding? Are there any general skills (e.g. writing math clearly, understanding what the problem is asking, etc.) that you would like to practice?

Week 1	Build a Network
Week 2	Actively Engage
Week 3-4	Plan & Prepare

## Goal: Make Adjustments

## Reflection: Making Adjustments

*“As I am working through the exam, I [realize] that I have a conceptual understanding of certain topics, but I need to **do more practice problems rather than [just reviewing] the textbook** and my notes. Enough exposure to different types of problems would have forced me to apply [what I know] in unfamiliar situations.”*

*“I learned that I **really need to [practice my] algebra skills**. I often don't feel comfortable rearranging equations and I could easily improve my calculus ability by improving my algebra skills.”*

*I [learned that I had] misunderstood how to apply some of the concepts. I found it helpful to redo the questions I had gotten wrong because I was able to **slowly break down what the problem was asking [and approach] it in steps** as opposed to feeling intimidated by it and not knowing where to start.*

Week 11	Exam 2
Week 12	Make Adjustments
Week 13	
Week 14	Reflect & Grow

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5 pts

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Week 9-10	Plan & Prepare
Week 11	Exam 2
Week 12	Make Adjustments
Week 13	
Week 14	Reflect & Grow

## Goal: Review and Practice

We find that successful Math 115 students make time to review what they learned in class and do some extra practice when they encounter a challenging topic.

### Strategy 1: Review your notes from class

Why not give it a try? Set aside some time either right after class, right before class, or perhaps at the end of the week to review the notes you took in class. Rather than just reading them, you may want to stop and work through some example problems on your own. Make sure to ask yourself questions as you go: why did we choose that test? Why couldn't we use this theorem instead? What is the idea behind this rule? If you are unsure of an answer to any question you raise, you may want to drop by peer tutoring or office hours to clarify!

### Strategy 2: Do some extra practice

As you review your notes from class and work on your problem set, are there any topics that you are struggling with or find particularly challenging? If so, try a few extra practice problems. The textbook has plenty of problems to choose from, and if you are not sure where to start, ask your instructor or a peer tutor!

You will be asked to reflect on how these strategies worked for you in next week's Explore.

Week 1	Build a Network
Week 2	Actively Engage
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Week 13	
Week 14	Reflect & Grow

**Goal: Plan and Prepare**

We find that successful Math 115 students begin thinking about how they will prepare for exams well in advance. They plan out how they will manage their time and adapt their plan as necessary. Let's return to the following strategy:

**Strategy: Create a Two-Week Study Plan**

Our second exam is on 4/7 -- that's just under 2 weeks after we get back from break! Now is a great time to begin deciding how you will manage your time, so why not give it another try? You may want to start with your study plan from the first exam and make any necessary changes and improvements, or you may want to start fresh -- it's totally up to you!

<b>Question 1</b>	<b>5 pts</b>
<p>Brainstorm some ideas for your study plan here. Are there any parts of your study plan from Exam 1 that you found particularly helpful that you want to try again? Are you planning to make any changes to your study plan this time around?</p>	

Week 1	Build a Network
Week 2	Actively Engage
Week 3-4	Plan & Prepare
Week 5	Exam 1
Week 6	Make Adjustments
Week 7	Review & Practice
Week 8	
Week 9-10	Plan & Prepare
Week 11	Exam 2
Week 12	Make Adjustments
Week 13	
Week 14	Reflect & Grow

## Goal: Reflect and Grow

Now that we are almost done with the semester, let's **reflect** on the learning strategies we have developed and think about how we can continue to **grow** as learners.

### Question 1

3 pts

Which learning strategies helped you improve as a learner throughout the semester? What would you like to work on in the future to continue improving as a learner?

Week 1	Build a Network
Week 2	Actively Engage
Week 3-4	Plan & Prepare

## Goal: Reflect and Grow

Now that we are almost done with the semester, let's **reflect** on the learning strategies we have developed and think about how we can continue to **grow** as learners.

*“Thinking about a test two weeks in advance was extremely helpful, even if I wasn't actively studying the entire time, it helped me a lot with time management.”*

*“Reflecting on my studying habits [motivated me to] build a routine that would help me in the long run.”*

*“I really struggled to build strong habits in this course. When a class builds on itself, it is **hard to catch up once you fall behind**, so I need to build effective routines earlier.”*

*“Going back through the notes and doing the exercises [after] each lesson really helped me learn the material better.”*

*“Actively [engaging] in class and making a consistent effort to learn the material little by little”.*

*Doing lots of **practice problems**, instead of memorizing formulas or looking at answers [is key].*

*[Collaboration] increased my confidence and performance. [It also] improved my attitude towards math.*

Week 14	Reflect & Grow
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# Think – Pair – Share:

What are the biggest challenges of teaching metacognition?

- Mixed buy-in
- Wide range of experiences
- Balancing guidance/scaffolding vs. student autonomy
- Time is limited



# Adapting to Proof-Based Courses



- Early semester reflection and feedback
- Use “Think-aloud” during proofs
  - Instructor models how they first encounter a new problem: speaking aloud their initial impressions, the preliminary questions they ask themselves to get started, where/when they get stuck and how to persist
  - Have students practice “Think-aloud” in pairs or small groups
    - Before beginning a proof, after completing a proof, or at a sticking point in a proof
    - In response to a pre-class reading
- Post assessment reflections (e.g. exam wrappers)

What other ideas do you have?

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# Adapting to

- Early semester reflection and
- Use “Think-aloud” during pro
- Instructor models how they, r  
preliminary questions they as
- Have students practice “Thin
- Before beginning a proof,
- In response to a pre-class
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What other ideas do yo

## Mid-semester Feedback -- Math 232

Thank you for taking the time to complete this survey.

The beginning of proof-based mathematics is a challenging transition for everyone -- it takes time, practice, resilience, and often new perspectives and strategies for learning to adjust effectively.

What has been the biggest challenge for you as you make this transition to proof-based math?

Your answer

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What is one strategy you would like to try to improve your learning when it comes to proofs?

Your answer

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## Pre-class -- Section 14-15 -- Equivalence Relations

## Question 1

Which of the following properties must an equivalence relation satisfy?

(A) reflexive	Correct answer	33.33%
(B) irreflexive		-33.33%
(C) symmetric	Correct answer	33.33%
(D) antisymmetric		-33.33%
(E) transitive	Correct answer	33.34%

Automated feedback



### Open-ended Question:

In the reading, you read a careful proof of Theorem 15.5 that states that congruence modulo  $n$  is an equivalence relation. What was the most challenging or confusing part of the proof?

**Summarize your thoughts in 2-3 sentences in the "Additional Content" box below and be ready to discuss in class.**

# Adapt

- Early semester
- Use “Think-aloud”
- Instructor modeling preliminary
- Have students
  - Before class
  - In response
- Post assessment

# What other

# Adapt

- Early semester
- Use “Think-aloud”
- Instructor re-examine preliminary
- Have students
  - Before class
  - In response
- Post assessment

What other

## How would you describe your study schedule:

- \_\_\_\_ I studied regularly each week
- \_\_\_\_ I started studying 1-2 weeks before the exam
- \_\_\_\_ I started studying 1-2 days before the exam
- \_\_\_\_ I divided up my studying into several short sessions
- \_\_\_\_ I did most of my studying in one large session
- \_\_\_\_ Other: \_\_\_\_\_

## What percentage of your studying was focused on:

- Making my cheat sheet \_\_\_\_\_
- Definitions & theorems \_\_\_\_\_
- Computational algorithms \_\_\_\_\_
- Proofs \_\_\_\_\_

## How much time did you spend on each study strategy:

- Skimming the textbook \_\_\_\_\_
- Carefully reading the textbook \_\_\_\_\_
- Reviewing my notes \_\_\_\_\_
- Practicing definitions & theorems \_\_\_\_\_
- Reworking old problems from class or problem sets \_\_\_\_\_
- Working additional problems \_\_\_\_\_
- Other (please explain below) \_\_\_\_\_

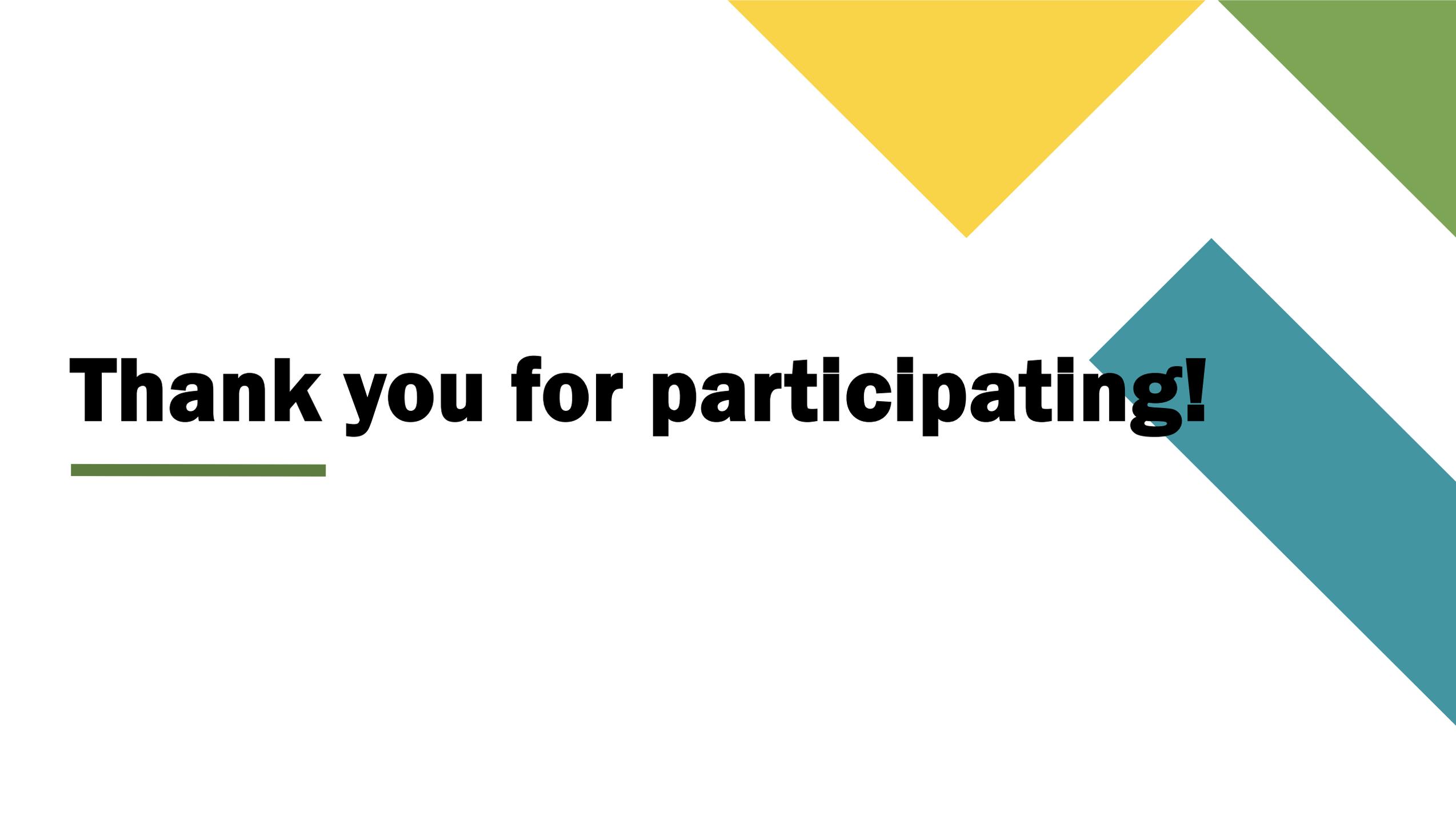
## What percentage of your studying was done

- Alone \_\_\_\_\_
- Collaborating with my peers \_\_\_\_\_
- Attending office hours or HOOT sessions \_\_\_\_\_

## Estimate the percentage of points you lost due to:

- Not remembering a definition or theorem \_\_\_\_\_
- Computational errors \_\_\_\_\_
- Not knowing how to solve a problem \_\_\_\_\_
- Not understanding a concept \_\_\_\_\_
- Getting stuck on a proof \_\_\_\_\_
- Mathematical writing errors \_\_\_\_\_
- Other (please explain below) \_\_\_\_\_

**Based on these estimates, what study strategies worked well for you? Is there anything you plan to do differently next time?**



**Thank you for participating!**

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# References

Cross, D. R., & Paris, S. G. (1988). Developmental and instructional analyses of children's metacognition and reading comprehension. *Journal of Educational Psychology*, 80(2), 131–142.

De Boer, H., Donker, A.S., Kostons, D.D., & Van der Werf, G.P. (2018). Long-term effects of metacognitive strategy instruction on student academic performance: A meta-analysis. *Educational Research Review*, 24, 98–115.

Desoete, A., & De Craene, B. (2019). Metacognition and mathematics education: An overview. *ZDM Mathematics Education*, 51, 565–575.

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Donker, A.S., de Boer, H., Kostons, D., Van Ewijk, C.D., & van der Werf, M.P. (2014). Effectiveness of learning strategy instruction

Guo, L. (2022). The effects of self-monitoring on strategy use and academic performance: A metaanalysis. *International Journal of Educational Research*, 112, 101939

Hacker, D. J., Dunlosky, J., & Graesser, A. C. (Eds.). (1998). *Metacognition in educational theory and practice*. Lawrence Erlbaum Associates.

# References

Halmo, S. M., Yamini, K. A, and Stanton, J. D. (2024). Metacognition and Self-Efficacy in Action: How First-Year Students Monitor and Use Self-Coaching to Move Past Metacognitive Discomfort During Problem Solving. *CBE—Life Sciences Education* 2024 23:2

Hattie, J. (2009). *Visible learning: A synthesis of over 800 meta-analyses relating to achievement*. New York, NY: Routledge.

Hein, I. M., Troost, P. W., Broersma, A., de Vries, M. C., Daams, J. G., & Lindauer, R. J. L. (2015). Why is it hard to make progress in assessing children's decision-making competence? *BMC Medical Ethics*, 16, 1.

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Huang, X., Bernacki, M. L., Kim, D., & Hong, W. (2022). Examining the role of self-efficacy and online metacognitive monitoring behaviors in undergraduate life science education. *Learning and Instruction*, 80, 101577

Jacobse, A. E., & Harskamp, E. G. (2012). Towards efficient measurement of metacognition in mathematical problem solving. *Metacognition Learning*, 7, 133–149.

Mevarech, Z. R., & Amrany, C. (2008). Immediate and delayed effects of meta-cognitive instruction on regulation of cognition and mathematics achievement. *Metacognition and Learning*, 3, 147–157.

# References

Muncer, G., Higham, P. A., Gosling, C. J., Cortese, S., Wood-Downie, H., & Hadwin, J. A. (2022). A meta-analysis investigating the association between metacognition and math performance in adolescence. *Educational Psychology Review*, 34(1), 301–334.

Ohtani, K., & Hisasaka, T. (2018). Beyond intelligence: A meta-analytic review of the relationship among metacognition, intelligence, and academic performance. *Metacognition and Learning*, 13(2), 179–212.

Pajares, F. (1996). Self-efficacy beliefs in academic settings. *Review of Educational Research*, 66(4), 543–578.

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Pintrich, P. R., & De Groot, E. V. (1990). Motivational and self-regulated learning components of classroom academic performance. *Journal of Educational Psychology*, 82, 33–40. <https://doi.org/10.1037/0022-0663.82.1.33>

Stanton, J. D., Sebesta, A. J., & Dunlosky, J. (2021). Fostering Metacognition to Support Student Learning and Performance. *CBE—Life Sciences Education*, 20(2)

Yu, C., Li, X., Wang, S., & Zhang, W. (2016). Teacher autonomy support reduces adolescent anxiety and depression: An 18-month longitudinal study. *Journal of Adolescence*, 49, 115–123.