Embodied Cognition: What is it? How does it involve mathematics?

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Online Seminar on Undergraduate Mathematics Education

October 11, 2022
Please do ...

Thank you, Google for all the fun images!
My Work

Mathematics

Cognition

Education
Outline

• Active Learning Definitions
• Playtime
• Embodied Cognition
• Sample Research Project
• Embodied Cognition & Active Learning
• Question & Answers
What is active learning?

- Classroom practices aimed at fostering student engagement that attends to the research-based idea that students learn best when they are engaged in their learning (Freeman et al., 2014)

- **Active learning consists of short course-related individual or small-group activities that all students in a class are called upon to do, alternating with instructor-led intervals in which student responses are processed and new information is presented** (Felder & Brent, 2009, p. 2)

- Inquiry based learning: **Students** engage deeply with meaningful mathematical tasks & collaboratively process mathematical ideas and **Instructors** inquire into student thinking & foster equity in their design and facilitation choices (Artigue & Blomhøj, 2013 as cited in Laursen & Rasmussen, 2019)
Time to Play

Volunteers Needed
Describe the notion of a limit.
Describe the notion of an integral.
Describe the notion of a normal subgroup.
What did you observe in each of the descriptions?
Embodied Cognition

Philosophy that describes reasoning as body-based stemming from experiences with the physical environment.
Historical Figures in Embodied Cognition

Jean Piaget’s
Stages of Cognitive Development

Sensorimotor Stage
The child begins to interact with the environment.
0-2

Preoperational Stage
The child begins to represent the world symbolically.
2-6 or 7

Concrete Operational Stage
The child learns rules such as conservation.
7-11 or 12

Formal Operational Stage
The adolescent can transcend the concrete and think about the future.
12–Adulthood

The principal goal of education in the schools should be creating men and women who are capable of doing new things, not simply repeating what other generations have done.” - Jean Piaget

A child’s play is not simply a reproduction of what he has experienced, but a creative reworking of the impressions he has acquired.

-Vygotsky
Vygotsky: Play & Movement

• Discovering language via play is an essential part of transition from other- to self-regulation.
• Imaginative play is essential to cognitive development.
• Play is such that the explanation for must always be that it is the imaginary... Imagination is a new formation that is not present in the consciousness... and represents a specifically human form of conscious activity. Like all functions of consciousness, it originally arises from action.
• Gesture represents an interpersonal connection between individuals. (1978, p. 56)
• "Young children name their drawings only after they have completed them; they need to see them before they can decide what they are." (1978, p. 28)
• “Movement is not separated from perception; the processes coincide almost exactly.” (1930, p. 15)
Varying Interpretations of Embodied Cognition

- Reasoning is based in perception and action \(\rightarrow\) *Mental Models* (Lakoff & Nuñez, 2000)
- Learning is doing (Husserl, 1907/1970a/1970b; Nemirovsky, 2012)
- Learning is moving in new ways (Abrahamson & Sanchez-Garcia, 2016)
  - students develop sensorimotor schemes as solutions to interaction problems
  - each scheme is oriented on an attentional anchor [...]
  - when *symbolic artifacts* are introduced into the arena, they may both mediate new affordances for students’ *motor-action* control and *shift* their *discourse* into explicit *mathematical re-visualization* of the environment (p. 203).
Embodied Cognition & Active Learning

Mental Models

Virtual Model

Physical World
Embodied cognition requires attending to utterances:
Embodied design applied to “tools whose operatory function is engineered specifically so as to . . . cultivate . . . the development of particular sensorimotor schemes as a condition for masterful control of the environment in accord with task demands” and thereby “come to ground mathematical concepts we want these students to learn” (Abrahamson & Bakker, 2016, p.5)
Forms of Embodiment and Student Learning
(Nathan, 2022)

<table>
<thead>
<tr>
<th>Grounding</th>
<th>Offloading</th>
<th>Cognitive-Sensorimotor Transduction</th>
<th>Participation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body form, movement &amp; perception</td>
<td>Pragmatic actions; Metaphors</td>
<td>Finger counting</td>
<td>Directed actions ←→ Reasoning</td>
</tr>
<tr>
<td>Gesture</td>
<td>Pointing; Iconic; Metaphoric</td>
<td>Pointing to imagined or real</td>
<td>Metaphors &amp; Gesture</td>
</tr>
<tr>
<td>Simulation</td>
<td>Instantiations</td>
<td>Enact movement on objects/diagrams</td>
<td>Cut, multiply, add for integration</td>
</tr>
<tr>
<td>Materialist Epistemology</td>
<td>Reification; Direct representations</td>
<td>Manipulatives; Models</td>
<td>Thought experiments; Functional actions</td>
</tr>
</tbody>
</table>
What is an ellipse?
Embodied activities activate our senses of hearing, seeing, & feeling/doing → learning
SAMPLE RESEARCH

In what ways does a mathematics educator, who is knowledgeable about embodiment, integrate embodiment to support students’ learning of abstract algebra concepts?
Data Collection

• Lead author attended every day of lecture, had weekly conversations with the instructor regarding the role of embodied cognition in her teaching practice

• Audio- and video- recorded six weeks of class

• Classroom data across these six weeks spanned: partitions and equivalence relations, cosets, homomorphisms, normal subgroups and factor groups, and the Fundamental Homomorphism Theorem (FHT).
Members of the research team (including instructor) **met twice a week to review all classroom data** divided into seven- and ten-minute segments.

**Initial coding**
We time-stamped each segment where the instructor integrated a form of embodiment and documented:

- type of gestures used
- instructor’s reflections on her intentional and unintentional forms of embodiment
- students’ classroom contributions.

**Summarized major themes:**
- regesturing students’ gestures
- delaying the use of formal language until students appeared familiar with everyday language or experienced physical encounters related to the formal concepts
- transforming students’ contributions into inscriptions

**Research team selected episodes** that exemplified the instructor’s integration of the various forms of embodiment:
- motivating partitions and equivalence classes through rock-dropping to partition $R \times R$
- **introducing cosets through weaves**
- acting out homomorphisms
- acting out conjugation towards the FHT

**Follow-up coding**
We used Nathan’s (2022) framework to clarify and enhance visibility of the instructor’s teaching in the embodied world.
- The two non-instructor research members coded the forms of embodiment and how they could support embodied learning
- Member-checking (Merriam & Tisdell, 2015)
Results
Projected a rock dropping into water during a class discussion on how to "split up" (partition) the cartesian plane.

Gestured to encourage a simulation of the live experience of a rock dropping into water.

"If Colin is in the same group as Lucas and Lucas is in the same group as Emily, then Colin is in the same group as Emily."

Rock Drop & Equivalence Relations
Weaves & Cosets

• Today, I want to start by simply training your eyes to look for patterns.

• What patterns if any do you notice?

• What do you see that is the same what do you see that is different?
“To me these sides look the same, you’ve got the green strands then the blue, then the darker blue (swept hands top to bottom), and then the two reds right here (swept hands left to right).” - Levi

“One has higher resolution, but the patterns are the same.” - Ian
<table>
<thead>
<tr>
<th>Simulation set up: students as elements</th>
<th>Function simulation to check onto and one-to-one</th>
<th>Homomorphism property simulation “exhaustive” strategy</th>
<th>FHT simulation “alternative” strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle Z_8, + \rangle$</td>
<td>$\langle Z_4, + \rangle$</td>
<td>$f: \langle Z_8, + \rangle \rightarrow \langle Z_4, + \rangle$</td>
<td>$f(a \ast b) = f(a) \ast f(b)$</td>
</tr>
</tbody>
</table>

| Margaret → 0 | Cathy → 0 | $f$ of Margaret you’re Cathy, $f$ of Greg you’re Jayden, $f$ of David you’re Jeffrey, $f$ of Jim you’re Nick, $f$ of Cam you’re Cathy, $f$ of Julie you’re Jayden, $f$ of Patrick you’re Jeffrey, $f$ of Keith you’re Nick | - Operate then map | - Finding Kernel $K = \{\text{Margaret, Cam}\}$ $K$ operated by everything in $Z_8$ to partition $Z_8$ into cosets of $g_iH$ \( i \in \mathbb{Z} \) $g_j \in g_iH \iff g_j \in g_iH \}$ Conjugation on the kernel Isomorphism from cosets as a group to $\langle Z_4, + \rangle$ |
| Greg → 1    | Jayden → 2 | $f$ of Greg you’re Jayden, $f$ of David you’re Jeffrey, $f$ of Jim you’re Nick, $f$ of Cam you’re Cathy, $f$ of Julie you’re Jayden, $f$ of Patrick you’re Jeffrey, $f$ of Keith you’re Nick | - Operate then map | - Finding Kernel $K = \{\text{Margaret, Cam}\}$ $K$ operated by everything in $Z_8$ to partition $Z_8$ into cosets of $g_iH$ \( i \in \mathbb{Z} \) $g_j \in g_iH \iff g_j \in g_iH \}$ Conjugation on the kernel Isomorphism from cosets as a group to $\langle Z_4, + \rangle$ |
| Jim → 3     | Nick → 3  | $f$ of David you’re Jeffrey, $f$ of Jim you’re Nick, $f$ of Cam you’re Cathy, $f$ of Julie you’re Jayden, $f$ of Patrick you’re Jeffrey, $f$ of Keith you’re Nick | - Operate then map | - Finding Kernel $K = \{\text{Margaret, Cam}\}$ $K$ operated by everything in $Z_8$ to partition $Z_8$ into cosets of $g_iH$ \( i \in \mathbb{Z} \) $g_j \in g_iH \iff g_j \in g_iH \}$ Conjugation on the kernel Isomorphism from cosets as a group to $\langle Z_4, + \rangle$ |
| Cam → 4     |           | $f$ of David you’re Jeffrey, $f$ of Jim you’re Nick, $f$ of Cam you’re Cathy, $f$ of Julie you’re Jayden, $f$ of Patrick you’re Jeffrey, $f$ of Keith you’re Nick | - Operate then map | - Finding Kernel $K = \{\text{Margaret, Cam}\}$ $K$ operated by everything in $Z_8$ to partition $Z_8$ into cosets of $g_iH$ \( i \in \mathbb{Z} \) $g_j \in g_iH \iff g_j \in g_iH \}$ Conjugation on the kernel Isomorphism from cosets as a group to $\langle Z_4, + \rangle$ |
| Julie → 5   |           | $f$ of David you’re Jeffrey, $f$ of Jim you’re Nick, $f$ of Cam you’re Cathy, $f$ of Julie you’re Jayden, $f$ of Patrick you’re Jeffrey, $f$ of Keith you’re Nick | - Operate then map | - Finding Kernel $K = \{\text{Margaret, Cam}\}$ $K$ operated by everything in $Z_8$ to partition $Z_8$ into cosets of $g_iH$ \( i \in \mathbb{Z} \) $g_j \in g_iH \iff g_j \in g_iH \}$ Conjugation on the kernel Isomorphism from cosets as a group to $\langle Z_4, + \rangle$ |
| Patrick → 6 |           | $f$ of David you’re Jeffrey, $f$ of Jim you’re Nick, $f$ of Cam you’re Cathy, $f$ of Julie you’re Jayden, $f$ of Patrick you’re Jeffrey, $f$ of Keith you’re Nick | - Operate then map | - Finding Kernel $K = \{\text{Margaret, Cam}\}$ $K$ operated by everything in $Z_8$ to partition $Z_8$ into cosets of $g_iH$ \( i \in \mathbb{Z} \) $g_j \in g_iH \iff g_j \in g_iH \}$ Conjugation on the kernel Isomorphism from cosets as a group to $\langle Z_4, + \rangle$ |
| Keith → 7   |           | $f$ of David you’re Jeffrey, $f$ of Jim you’re Nick, $f$ of Cam you’re Cathy, $f$ of Julie you’re Jayden, $f$ of Patrick you’re Jeffrey, $f$ of Keith you’re Nick | - Operate then map | - Finding Kernel $K = \{\text{Margaret, Cam}\}$ $K$ operated by everything in $Z_8$ to partition $Z_8$ into cosets of $g_iH$ \( i \in \mathbb{Z} \) $g_j \in g_iH \iff g_j \in g_iH \}$ Conjugation on the kernel Isomorphism from cosets as a group to $\langle Z_4, + \rangle$ |
Cognitive sensorimotor transduction, grounding, and participation

Operate then map four-person body formation

Map then operate five-person body formation

Types of embodiment in teaching abstract algebra concepts

Body form

Simulation

Integrated to support:

Embodied learning

Communities of practice

(grounding, offloading, cog. sensorimotor transduction, and participation)
Melody or Connor (the Kernel) "is going to be in the middle and is going to be operated on the left-hand side by the inverse and on the right-hand side by the element."

Conjugation & Normal Subgroup
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<tr>
<td>Simulation</td>
<td>Created physical referents to ground factor group and FHT concepts</td>
<td>Scaffolded function, hom, and FHT simulations to control amount students had to focus on at a given moment</td>
<td>Simulation provided space for students to develop their own shared body language</td>
</tr>
<tr>
<td></td>
<td>FHT simulation offloaded computational stress of brute force strategies</td>
<td>Facilitated pathways between movements embedded in simulation and abstract concepts</td>
<td>Led to community of practice roles (i.e., translators, body formation builders, etc.)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Included others who may have not been fluent in the formal language</td>
</tr>
</tbody>
</table>
Following scaffolded simulations both “exhaustive and alternative” the instructor ended class with a Paddy Paper (materials) and *Group Explorer Activity*

transition from embodied to formal-symbolic language

David realized that “looking at this [Cayley table for $D_4$] … we noticed the colors … you got your warm and your cool colors here. And earlier in class, …, we were talking about, oh, the cool ones are almost subgroups, but they’re not because they don’t contain the identity. … **now we are realizing that if they have the same number of things like they are kind of subsets, but not subgroups [cosets]. Then that’s how you construct a homomorphism to your subgroups or subsets [cosets]. And then this is the homomorphism, and the image is isomorphic to $S_2$**
Teachers’ Revoicing & Regesturing (Alibali & Nathan, 2019)

- Repeated students’ turns at talk and added gestures that indicated specific referents such as inscriptions.
- Created gestures, without speaking, to correspond with the students’ verbiage
  - Gestures referred to the students’ referents which were spoken without gesture or spoken but with gestures that are distant from their [spoken] referents
- Appeared to highlight students’ verbal utterances, to correct students’ verbiage, or to help develop a shared understanding in the classroom.
Discussion & Implications: Revoicing & Regesturing

• The instructor used simulation, rather than diagrams or materials, along with gesture:
  ✓ Closed the distance between referents of the instructors' gesture and students because students were the referents of the instructors’ gestures

• Facilitate communities of practice:
  ✓ Roles and dividing up the workload (scribes, body formation builders, planners, embodied to formal translators, etc.)-acting out homomorphism episode
  ✓ Students worked together to perform mathematical operations (mapping and conjugation)

• Positioned students' contributions and legitimate:
  ✓ Students developed their own sign (standing beside each other and holding up fingers to count) for conjugation

• Implications for teaching practice:
  ✓ Our work provides examples of how educators might integrate various forms of embodiment into their classrooms to support learning, students’ contributions, and communities of practice, even if the course is abstract.
Inquiry based learning: **Students** engage deeply with meaningful mathematical tasks & collaboratively process mathematical ideas and **Instructors** inquire into student thinking & foster equity in their design and facilitation choices (Artigue & Blomhøj, 2013 as cited in Laursen & Rasmussen, 2019)

**How do embodied activities attend to:**

- Student engagement
- Student collaborations
- Teacher inquiry
- Teacher fostering equitable teaching
Thank You!