Embodied Cognition: What is it? How does it involve mathematics?

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Online Seminar on Undergraduate Mathematics Education

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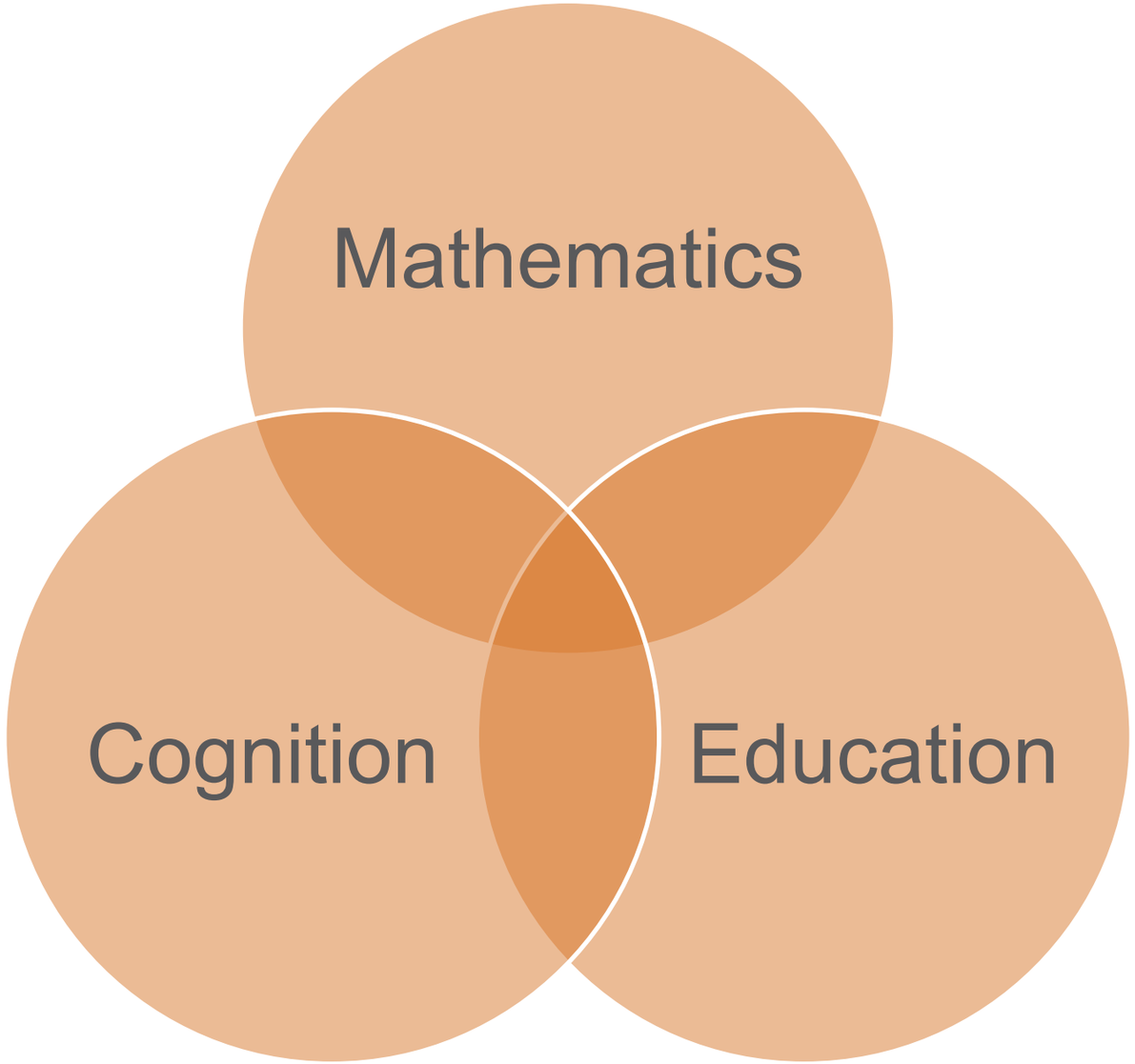
MAA
MATHEMATICAL ASSOCIATION OF AMERICA

Colorado State University

Please do …

Thank you, Google for all the fun images!

My Work



Outline

- Active Learning Definitions
- Playtime
- Embodied Cognition
- Sample Research Project
- Embodied Cognition & Active Learning
- Question & Answers

What is active learning?

- Classroom practices aimed at fostering student engagement that attends to the research-based idea that students learn best when they are engaged in their learning (Freeman et al., 2014)
- *Active learning consists of short course-related individual or small-group activities that all students in a class are called upon to do, alternating with instructor-led intervals in which student responses are processed and new information is presented* (Felder & Brent, 2009, p. 2)
- Inquiry based learning: **Students** engage deeply with meaningful mathematical tasks & collaboratively process mathematical ideas and **Instructors** inquire into student thinking & foster equity in their design and facilitation choices (Artigue & Blomhøj, 2013 as cited in Laursen & Rasmussen, 2019)

Time to Play

Volunteers Needed

Describe the notion of a limit.

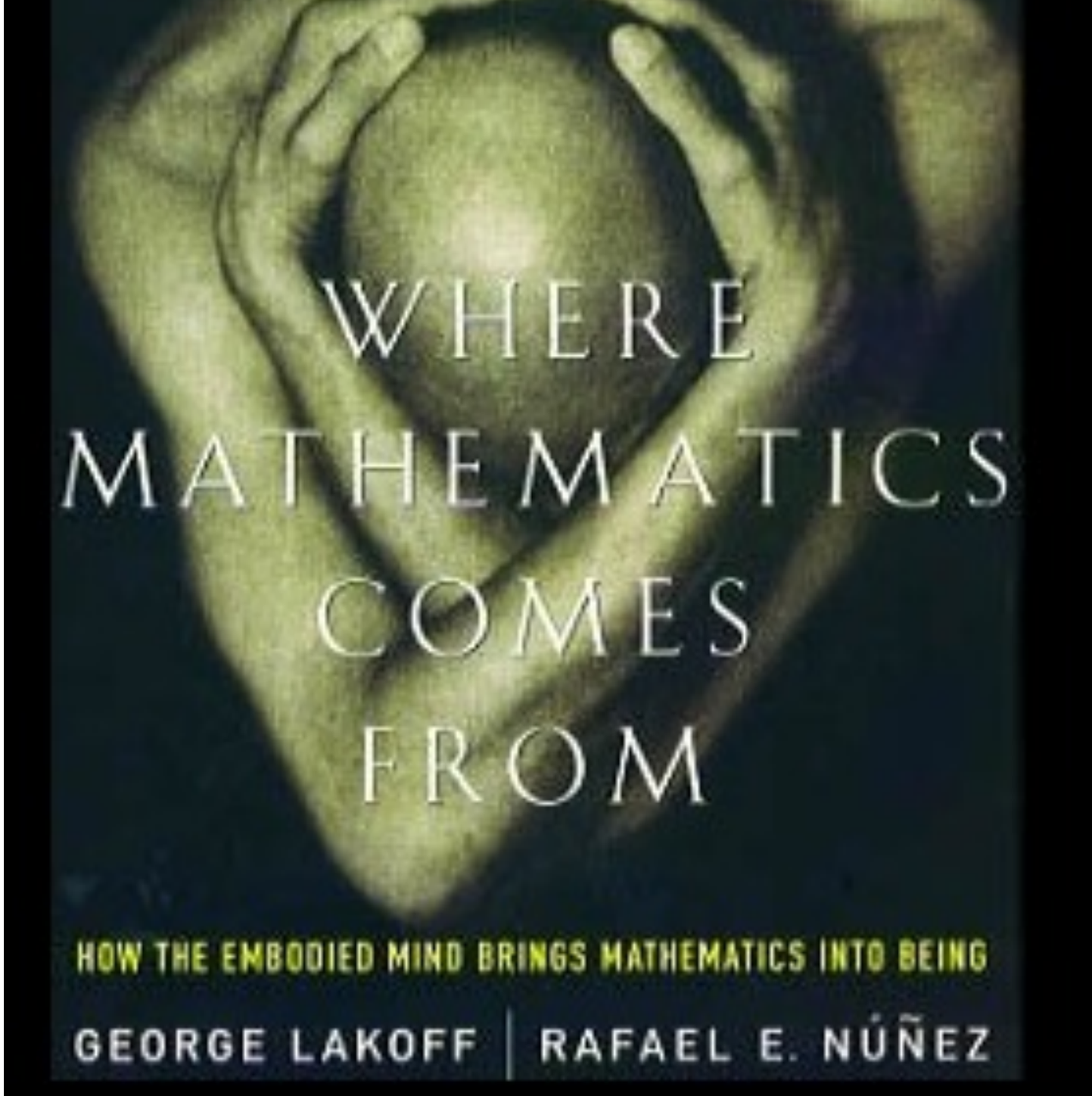
Describe the notion of an integral.

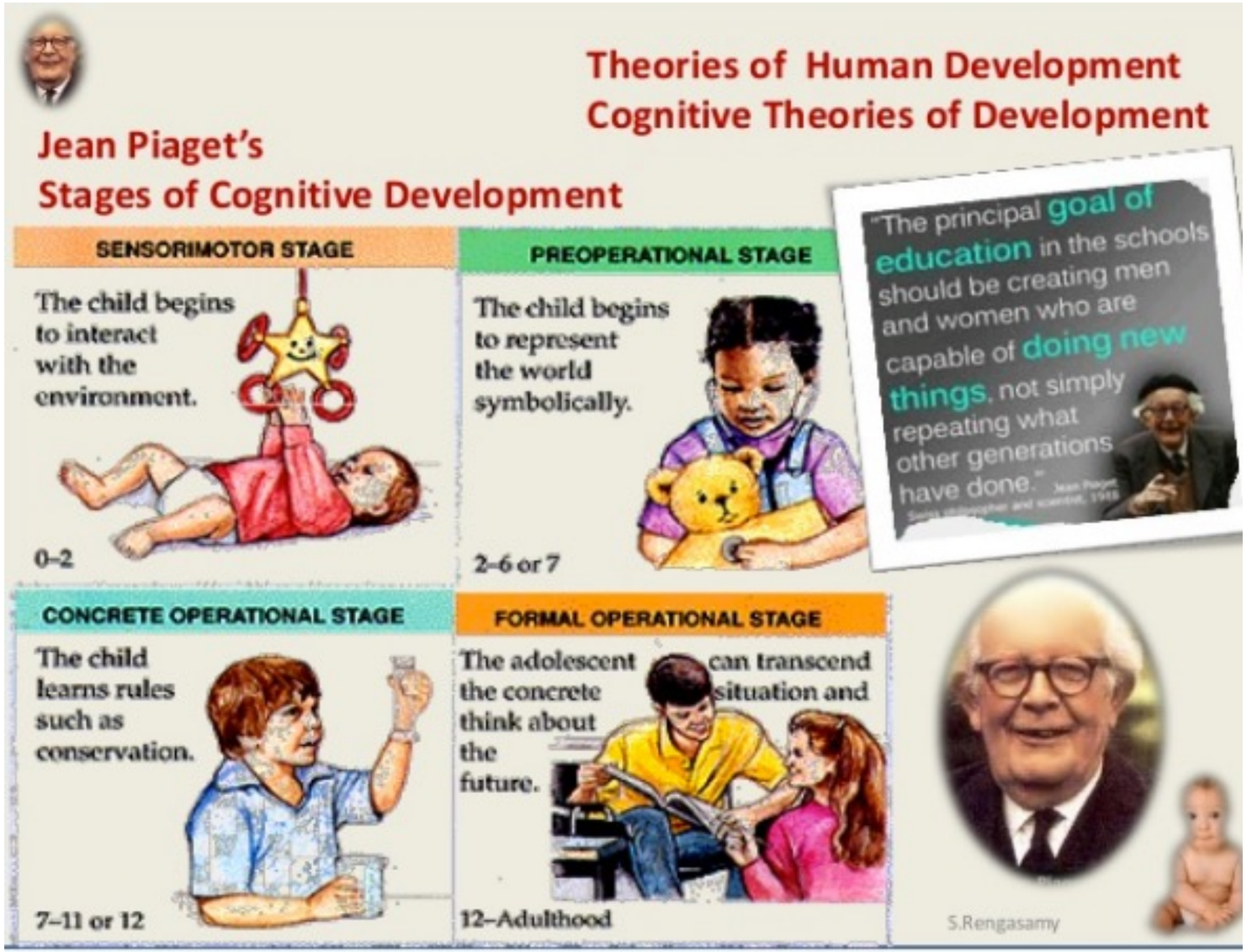
Describe the notion of a normal subgroup.

What did you observe in each of the descriptions?

Embodied Cognition

Philosophy that describes reasoning as body-based stemming from experiences with the physical environment





Historical Figures in Embodied Cognition

Vygotsky: Play & Movement

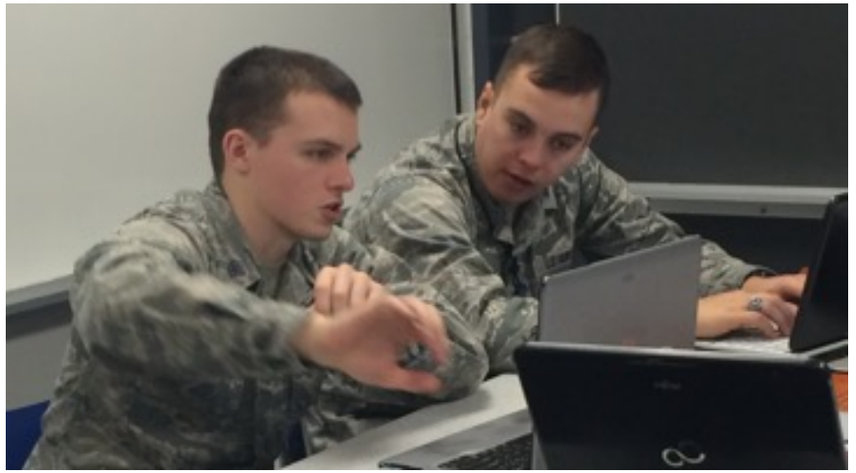
- **Discovering language via play** is an essential part of transition from other- to self-regulation
- **Imaginative play** is essential to cognitive development
- **Play** is such that the explanation for must always be that it is the **imaginary**… Imagination is a new formation that is not present in the **consciousness**...and represents a specifically human form of conscious activity. Like all functions of consciousness, it originally arises from **action**.
- **Gesture** represents an **interpersonal connection** between individuals**.** (1978, p. 56)
- ”Young children name their drawings only after they have completed them; they **need to see** them before they can decide what they are.” (1978, p. 28)
- “**Movement** is not separated from **perception**; the processes coincide almost exactly.” (1930, p. 15)

Varying Interpretations of Embodied Cognition

- Reasoning is based in perception and action → *Mental Models* (Lakoff & Nuñez, 2000)
- Learning is doing (Husserl, 1907/1970a/1970b; Nemirovsky, 2012)
- Learning is moving in new ways (Abrahamson & Sanchez-Garcia, 2016)
  - students develop sensorimotor schemes as solutions to interaction problems
  - each scheme is oriented on an attentional anchor […]
  - when **symbolic artifacts** are introduced into the arena, they may both mediate new affordances for students' **motor-action** control and **shift** their **discourse** into explicit **mathematical re-visualization** of the environment (p. 203).

Embodied Cognition & Active Learning

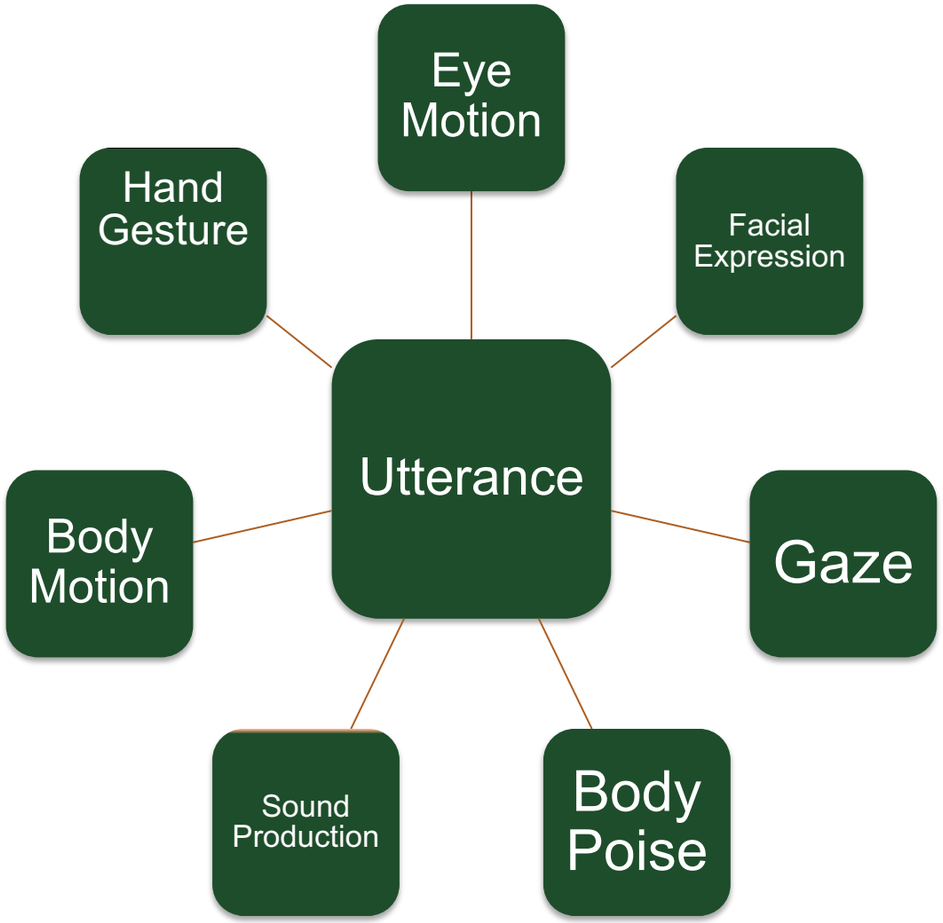
Mental Models



Virtual Model

Physical World

Embodied cognition requires attending to *utterances:*



Embodied design applied to “tools whose operatory function is engineered specifically so as to . . . cultivate . . . the development of particular sensorimotor schemes as a condition for masterful control of the environment in accord with task demands” and thereby “come to ground mathematical concepts we want these students to learn” (Abrahamson & Bakker, 2016, p.5)

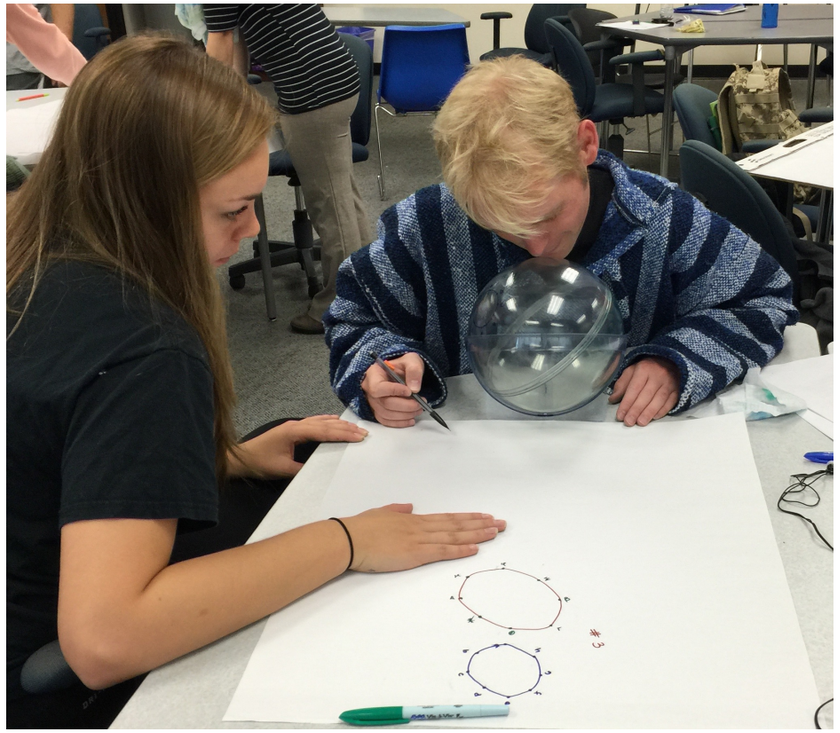
Forms of Embodiment and Student Learning

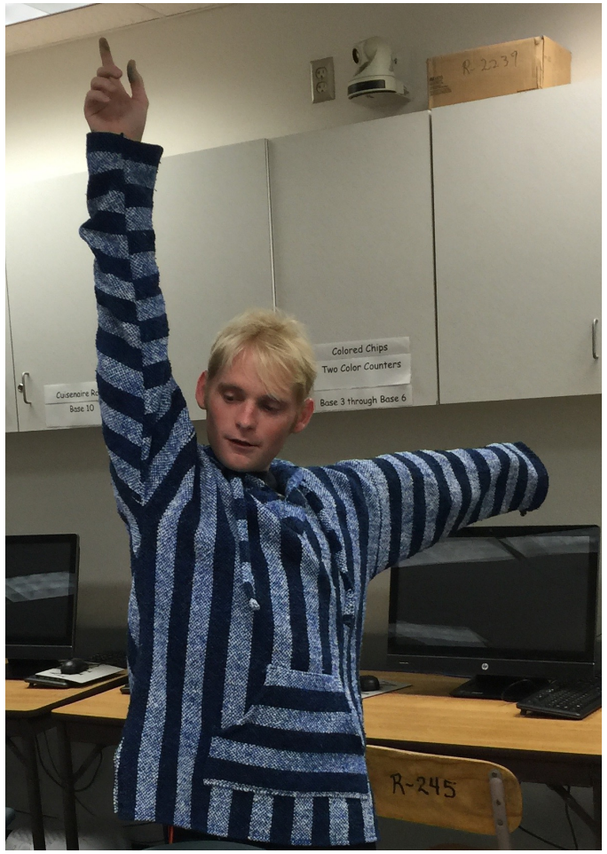
(Nathan, 2022)

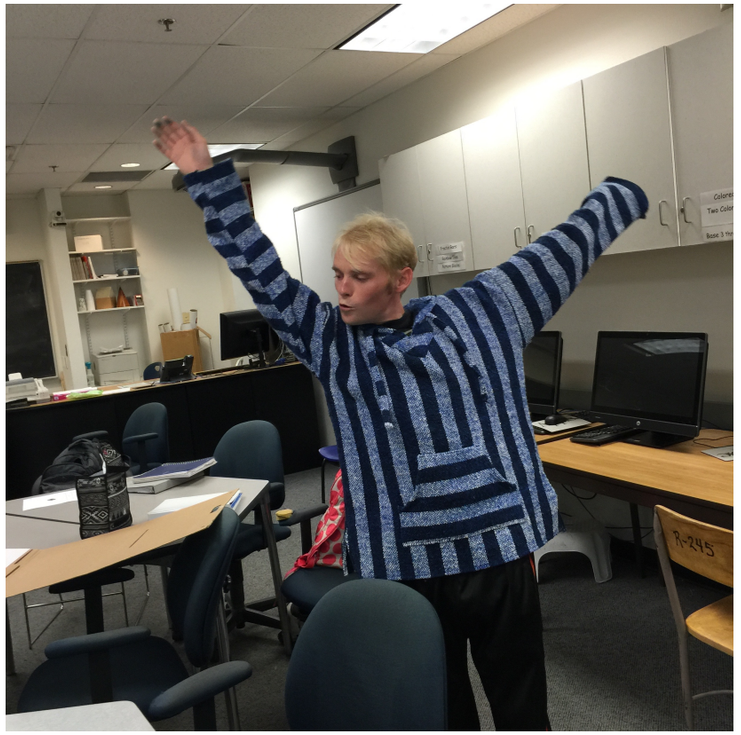
	Grounding	Offloading	Cognitive-Sensorimotor Transduction	Participation
Body form, movement & perception	Pragmatic actions; Metaphors	Finger counting	Directed actions ←→Reasoning	Communities of practice
Gesture	Pointing; Iconic; Metaphoric	Pointing to imagined or real	Metaphors & Gesture	Turn taking; Collaborative
Simulation	Instantiations	Enact movement on objects/diagrams	Cut, multiply, add for integration	Participatory simulation
Materialist Epistomology	Reification; Direct representations	Manipulatives; Models	Thought experiments; Functional actions	Groups/Teams

What is an ellipse?

Embodied Cognition Psychology: Use Your Body to Change Your Mind







Embodied activities **activate** our senses of hearing, seeing, & feeling/doing → **learning**

SAMPLE RESEARCH

In what ways does a mathematics educator, who is knowledgeable about embodiment, integrate embodiment to support students' learning of abstract algebra concepts?

Data Collection

- Lead author attended every day of lecture, had weekly conversations with the instructor regarding the role of embodied cognition in her teaching practice
- Audio- and video- recorded six weeks of class
- Classroom data across these six weeks spanned: partitions and equivalence relations, cosets, homomorphisms, normal subgroups and factor groups, and the Fundamental Homomorphism Theorem (FHT).

Members of the research team (including instructor) **met twice a week to review all classroom data** divided into seven- and ten-minute segments.

Initial coding

We time-stamped each segment where the instructor integrated a form of embodiment and documented:

- type of gestures used
- instructor's reflections on her intentional and unintentional forms of embodiment
- students' classroom contributions.

Summarized major themes:

- regesturing students' gestures
- delaying the use of formal language until students appeared familiar with everyday language or experienced physical encounters related to the formal concepts
- transforming students' contributions into inscriptions

Research team selected episodes that exemplified the instructor's integration of the various forms of embodiment:

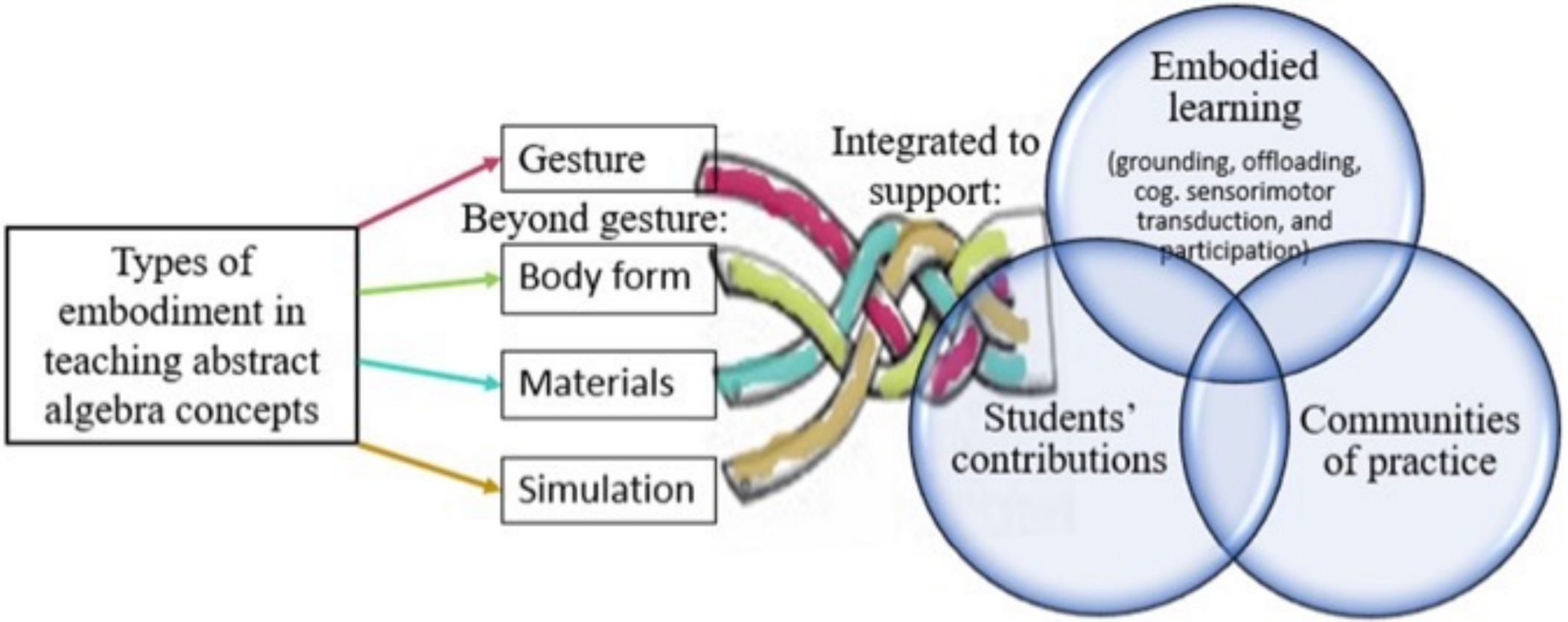
- motivating partitions and equivalence classes through rock-dropping to partition $R \times R$
- **introducing cosets through weaves**
- **acting out homomorphisms**
- acting out conjugation towards the FHT

Follow-up coding

We used Nathan's (2022) framework to clarify and enhance visibility of the instructor's teaching in the embodied world.

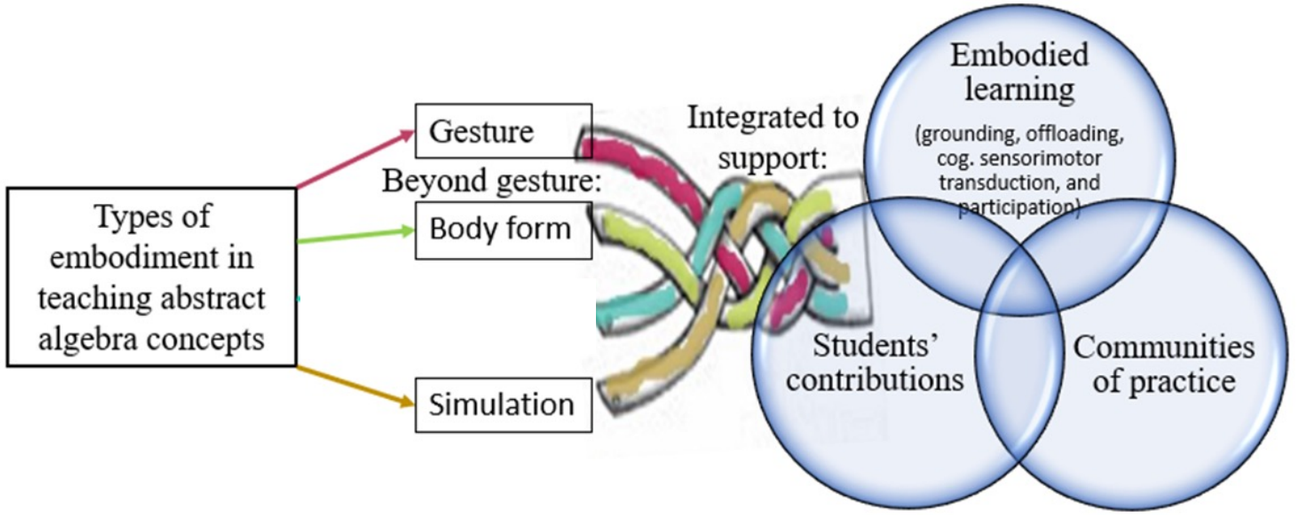
- The two non-instructor research members coded the forms of embodiment and how they could support embodied learning
- Member-checking (Merriam & Tisdell, 2015)

Results


Types of embodiment in teaching abstract algebra concepts
Gesture
Beyond gesture:
Body form
Materials
Simulation
Integrated to support:
Embodied learning
(grounding, offloading, cog. sensorimotor transduction, and participation)
Students' contributions
Communities of practice

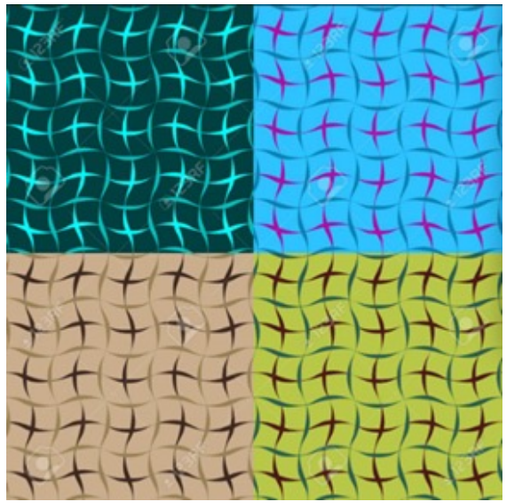
- Projected a rock dropping into water during a class discussion on how to "split up" (partition) the cartesian plane

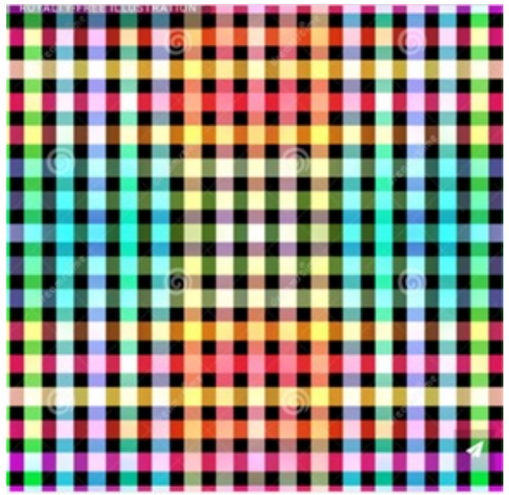
- Gestured to encourage a simulation of the live experience of a rock dropping into water
- "If Colin is in the same group as Lucas and Lucas is in the same group as Emily, then Colin is in the same group as Emily"



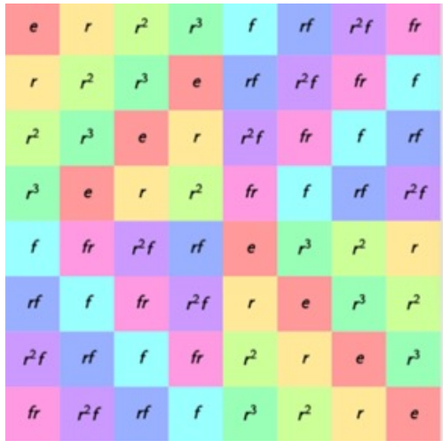
Rock Drop & Equivalence Relations

Weaves & Cosets





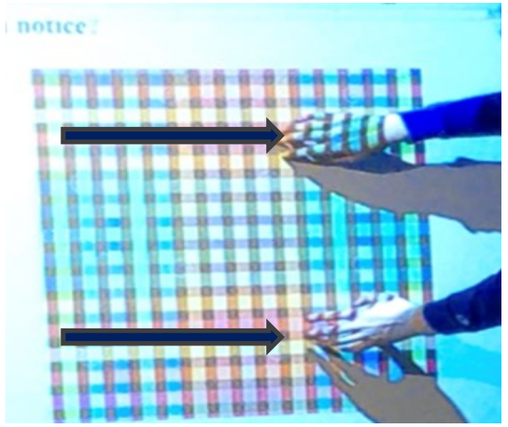
- Today, I want to start by simply training your eyes to look for patterns.
- What patterns if any do you notice?
- What do you see that is the same what do you see that is different?

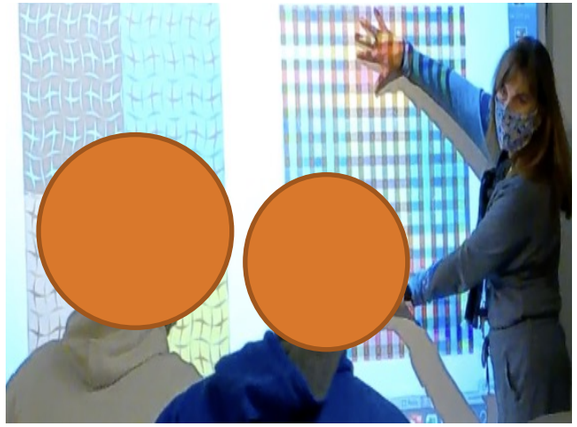


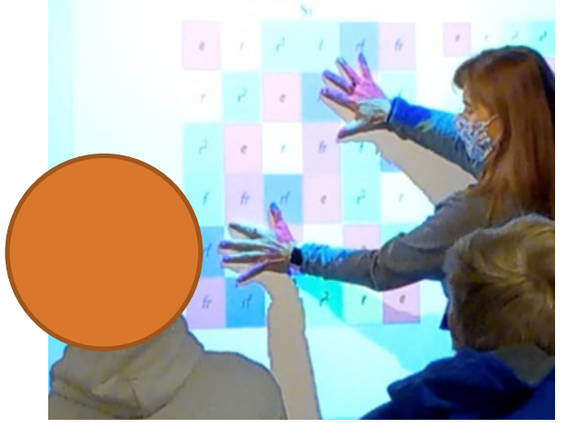
	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9	α_{10}	α_{11}	α_{12}
$(1) = \alpha_1$	1	2	3	4	5	6	7	8	9	10	11	12
$(12)(34) = \alpha_2$	2	1	4	3	6	5	8	7	10	9	12	11
$(13)(24) = \alpha_3$	3	4	1	2	7	8	5	6	11	12	9	10
$(14)(23) = \alpha_4$	4	3	2	1	8	7	6	5	12	11	10	9
$(123) = \alpha_5$	5	8	6	7	9	12	10	11	1	4	2	3
$(243) = \alpha_6$	6	7	5	8	10	11	9	12	2	3	1	4
$(142) = \alpha_7$	7	6	8	5	11	10	12	9	3	2	4	1
$(134) = \alpha_8$	8	5	7	6	12	9	11	10	4	1	3	2
$(132) = \alpha_9$	9	11	12	10	1	3	4	2	5	7	8	6
$(143) = \alpha_{10}$	10	12	11	9	2	4	3	1	6	8	7	5
$(234) = \alpha_{11}$	11	9	10	12	3	1	2	4	7	5	6	8
$(124) = \alpha_{12}$	12	10	9	11	4	2	1	3	8	6	5	7

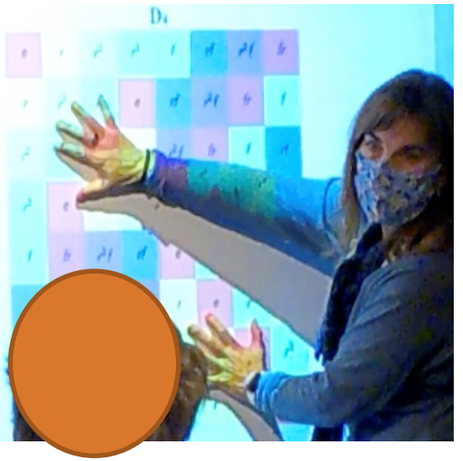
Participation

Ground and Offload



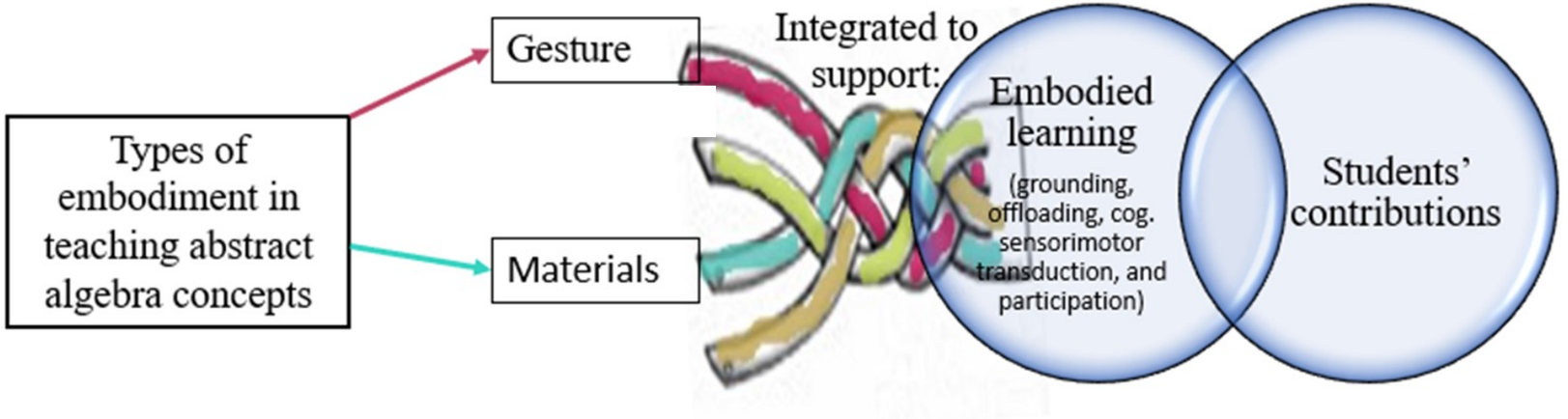






“To me these sides look the same, you’ve got the **green strands then the blue, then the darker blue** (swept hands top to bottom), and then the two reds **right here** (swept hands left to right).” - Levi

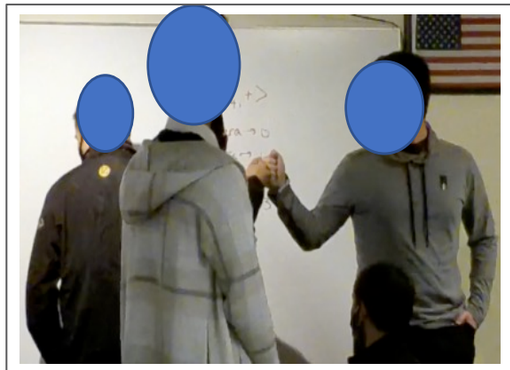
“One has higher resolution, but the patterns are the same.”-Ian

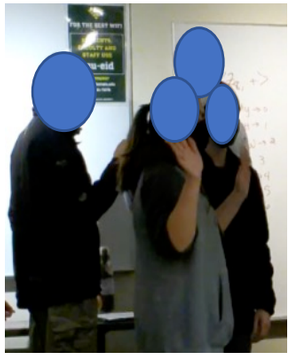


Simulation set up: students as elements		Function simulation to check onto and one-to-one	Homomorphism property simulation "exhaustive" strategy	FHT simulation "alternative" strategy
$\langle Z_8, + \rangle$	$\langle Z_4, + \rangle$	$f: \langle Z_8, + \rangle \rightarrow \langle Z_4, + \rangle$	$f(a * b) = f(a) * f(b)$	Prove $f: \langle Z_8, + \rangle \rightarrow \langle Z_4, + \rangle$ is a homomorphism
Margaret→ 0 Greg → 1 David → 2 Jim → 3 Cam → 4 Julie → 5 Patrick → 6 Keith → 7	Cathy → 0 Jayden → 1 Jeffrey → 2 Nick → 3	f of Margaret you're Cathy, f of Greg you're Jayden, f of David you're Jeffrey, f of Jim you're Nick, f of Cam you're Cathy, f of Julie you're Jayden, f of Patrick you're Jeffrey, f of Keith you're Nick	• Operate then map • Map then operate	• Finding Kernel K={Margaret, Cam} • K operated by everything in Z_8 to partition Z_8 into cosets • $g_j \in g_i H \Leftrightarrow g_j$ $g_i H$ • Conjugation on the kernel • Isomorphism from cosets as a group to $\langle Z_4, + \rangle$

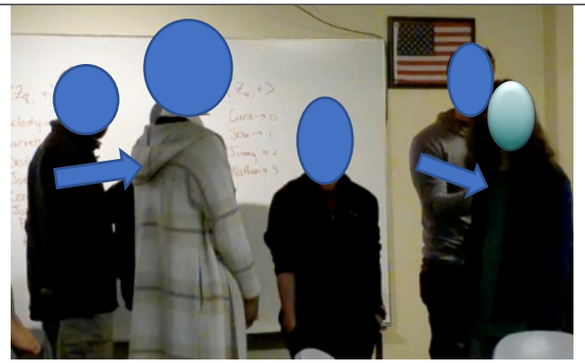
Acting out a Homomorphism

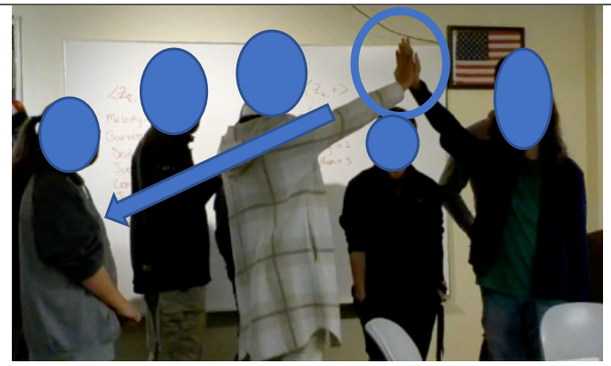
Cognitive sensorimotor transduction, grounding, and participation



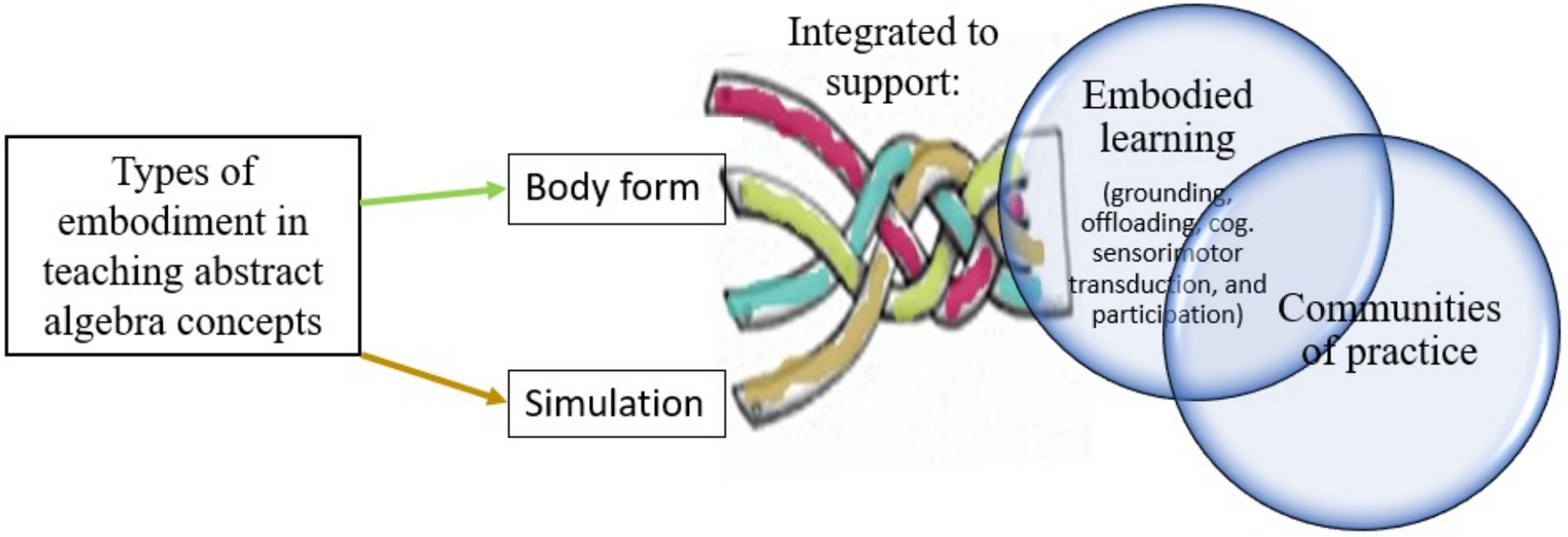


Operate then map four-person body formation



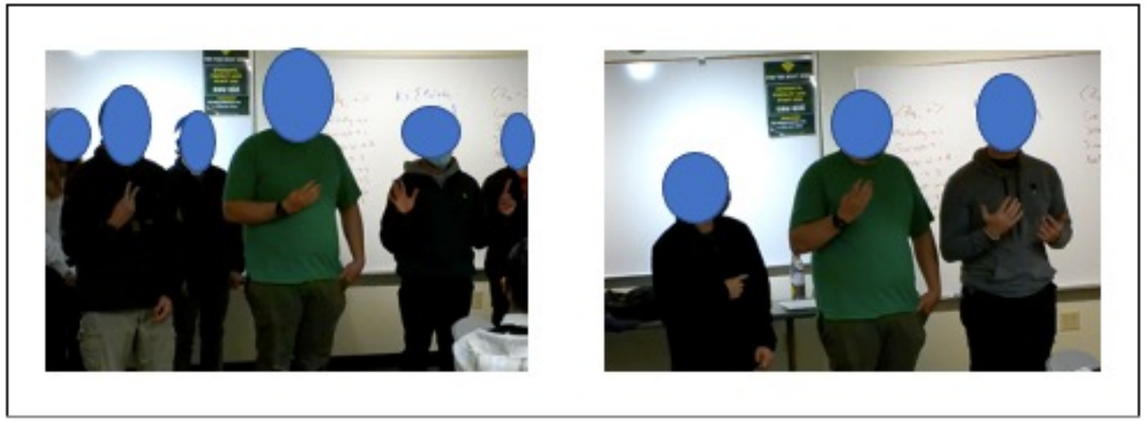


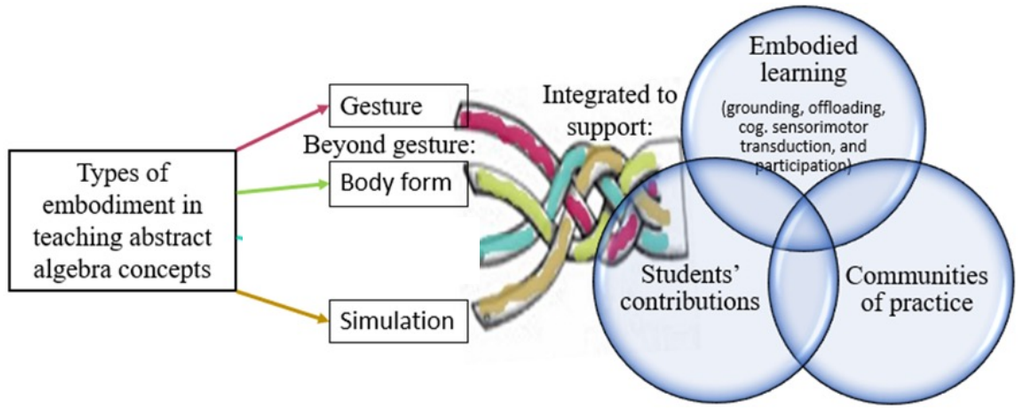
Map then operate five-person body formation



$$f: \langle \mathbb{Z}_8, + \rangle \rightarrow \langle \mathbb{Z}_4, + \rangle$$

Melody or Connor (the Kernel) "is going to be in the middle and is going to be operated on the left-hand side by the inverse and on the right-hand side by the element."



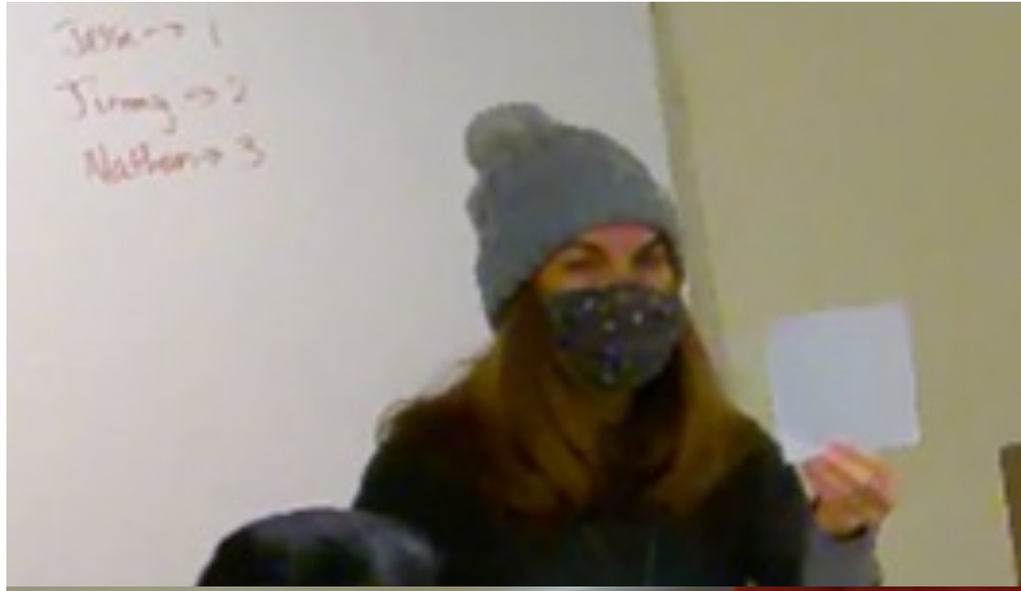


Conjugation & Normal Subgroup

	Grounding	Offloading	Cognitive-sensorimotor transduction	Participation
Simulation	Created physical referents to ground factor group and FHT concepts	Scaffolded function, hom, and FHT simulations to control amount students had to focus on at a given moment FHT simulation offloaded computational stress of brute force strategies	Facilitated pathways between movements embedded in simulation and abstract concepts	Simulation provided space for students to develop their own shared body language Led to community of practice roles (i.e., translators, body formation builders, etc.) Included others who may have not been fluent in the formal language

- Following scaffolded simulations both “exhaustive and alternative” the instructor ended class with a Paddy Paper (materials) and *Group Explorer* Activity
- transition from embodied to formal-symbolic language

David realized that “looking at this [Cayley table for D_4] … we noticed the colors … you got your warm and your cool colors here. And earlier in class, …, we were talking about, oh, the cool ones are almost subgroups, but they're not because they don’t contain the identity. … **now we are realizing that if they have the same number of things like they are kind of subsets, but not subgroups [cosets]. Then that’s how you construct a homomorphism to your subgroups or subsets [cosets]. And then this is the homomorphism, and the image is isomorphic to S_2**



Teachers' Revoicing & Regesturing (Alibali & Nathan, 2019)

- Repeated students' turns at talk and added gestures that indicated specific referents such as inscriptions.
- Created gestures, without speaking, to correspond with the students' verbiage
  - Gestures referred to the students' referents which were spoken without gesture or spoken but with gestures that are distant from their [spoken] referents
- Appeared to highlight students' verbal utterances, to correct students' verbiage, or to help develop a shared understanding in the classroom.

Discussion & Implications: Revoicing & Regesturing

- The instructor used simulation, rather than diagrams or materials, along with gesture:
  - ✓ Closed the distance between referents of the instructors' gesture and students because students were the referents of the instructors' gestures
- Facilitate communities of practice:
  - ✓ Roles and dividing up the workload (scribes, body formation builders, planners, embodied to formal translators, etc.)-acting out homomorphism episode
  - ✓ Students worked together to perform mathematical operations (mapping and conjugation)
- Positioned students' contributions and legitimate:
  - ✓ Students developed their own sign (standing beside each other and holding up fingers to count) for conjugation
- Implications for teaching practice:
  - ✓ Our work provides examples of how educators might integrate various forms of embodiment into their classrooms to support learning, students' contributions, and communities of practice, even if the course is abstract.

Embodied Cognition & Active Learning

Inquiry based learning: **Students** engage deeply with meaningful mathematical tasks & collaboratively process mathematical ideas and **Instructors** inquire into student thinking & foster equity in their design and facilitation choices (Artigue & Blomhøj, 2013 as cited in Laursen & Rasmussen, 2019)

How do embodied activities attend to:

- Student engagement
- Student collaborations
- Teacher inquiry
- Teacher fostering equitable teaching

Thank You!