Reshaping the

Calculus sequence for Engineering students

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SCHOOL OF ENGINEERING AND APPLIED SCIENCES

UNIVERSITY OF VIRGINIA

Engineering Math Project

Applied Math at UVA

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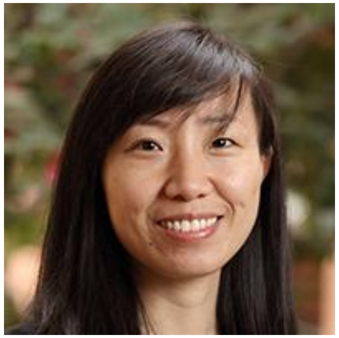

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Why - School of Engineering Setup

- Our school of engineering assumes that most (if not all) incoming students should begin with Calculus II (Calculus I is not a requirement).
- Most of our degree programs assume that students complete calculus by the end of their first year.
- In Fall 2017, less than 15% of our incoming students began with Calculus I. (~25% in Fall 2022).
- These students often attend summer courses in order to catch up, or they end up graduating later.
- These systemic forces result in an unhealthy dynamic that generates many negative feelings and poor decisions:

    1. incoming students that enroll in Calculus I are often afraid of falling behind their peers
    2. many incoming students that struggle with Calculus II will not move to Calculus I because they are also afraid of falling behind.

Why - Calculus Credit rush

- Many students have inadequate pre-calculus preparation
- Yet an increasing number of students are bypassing our calculus curriculum by obtaining calculus credit in high school.
- Enrollment in Calculus has declined despite a rise in students' enrollment in the School.
- (We feel strongly that high school calculus credits are not equivalent to our engineering-focused applied calculus courses.)
- Each semester we observe noticeable gaps between students of different levels.

These two factors, School of Engineering Setup and Calculus credit rush, have compelled us to reform the calculus curriculum at our school.

Objectives

Serve the needs of three distinct sets of first-year students.

Address gaps in skills and knowledge in all three sets.

Push back against college calculus avoidance in engineering students.

Allow all students to achieve mastery of basic skills needed for engineering education by the end of the first year.

Apply modern teaching techniques across the board (e.g. active learning and peer learners).

What

Engineering Math Tracks (2-semester sequences)

Core – For students who do not have confidence in their Calculus I background. Will cover the most important topics and skills necessary for later courses.

Regular – Equivalent to current Calculus II and Calculus III sequence.

Honors – For students who would currently be eligible to skip Calculus II and would like an enhanced experience that also fills the gaps.

Engineering Math Lab

Self-paced online modules consisting of videos and worksheets

Pre-Calculus

Cumulative assessment

Eliminate duplication of effort

What

Core Engineering Math I and II

- The core track is for those students whose calculus background is weak or lacking.
- These students have traditionally taken APMA 1090 (Calculus I) in the first semester, and needed three semesters to complete the calculus sequence.
- The core track begins with single variable Calculus I. By the end of the second semester, students will have completed multivariable Calculus III to an extent that is sufficient for moving on to more advanced classes.
- The Core Track provides the minimum necessary calculus foundation.
- Course structure with assisted active learning: smaller class size (instead of 45), pre-class reading & assignment, in class short lecture/review, in class collaborative group work with daily worksheets and constant TAs support, post-class daily assignments.
- Pre-calculus topics and certain selected topics are to be diverted to the Math Lab course.

What

Engineering Math I and II

- The regular track is for those students who have a good calculus background, having successfully completed single variable Calculus I (AP AB) in high school.
- The sequence for these students will be similar to those students who start with APMA 1110 (Calculus II) today.
- They will take two semesters of calculus that will begin with single variable Calculus II and conclude with multivariable Calculus III.
- This is a regular course (enrollment 45) with in class active learning, daily tasks, and weekly assignments/tests. TAs support limited to class activities.

What

Honors Engineering Math I and II

- The advanced track is for the students with the strongest math background.
- These students have successfully completed single variable Calculus II through an AP Calculus BC course (Score 5 and ready to take Calculus III).
- These students will take two semesters of calculus in which the content will be enhanced to cover some of the topics more thoroughly and to cover additional challenging topics.
- Class size 30, group activities on projects spanning few lectures (e.g. a week), where students will team up and work mostly autonomously, with a significant amount of self-directed learning. Minimal TA support.

What

Math Lab course - one semester

- The Math Lab will address the wide variability among our incoming students in their knowledge and skills in non-calculus areas. For example, topics like limits and determinants currently need to be reviewed in multiple courses.
- The Math Lab course will cover these topics in a self-paced environment. A module that includes a video lecture and practice worksheets will be offered for each topic.
- Modules may be completed throughout the semester, and students will be provided with several opportunities to complete the first semester Math Lab requirement by passing a comprehensive test.
- The most competent students will be able to pass the test and receive credit at the beginning of the semester. Since all students will need to demonstrate competency in these areas, time will be saved in other courses. These topics will no longer need to be reviewed multiple times in multiple courses.

First Implementation Step

Fall 2016 Honor Engineering Math I

How were students selected?

Self-Enrolled with college credit for Single Variable Calculus II or score 5 on the Advanced Placement Calculus BC exam. 26 students (6 females, 20 males).

Objectives?

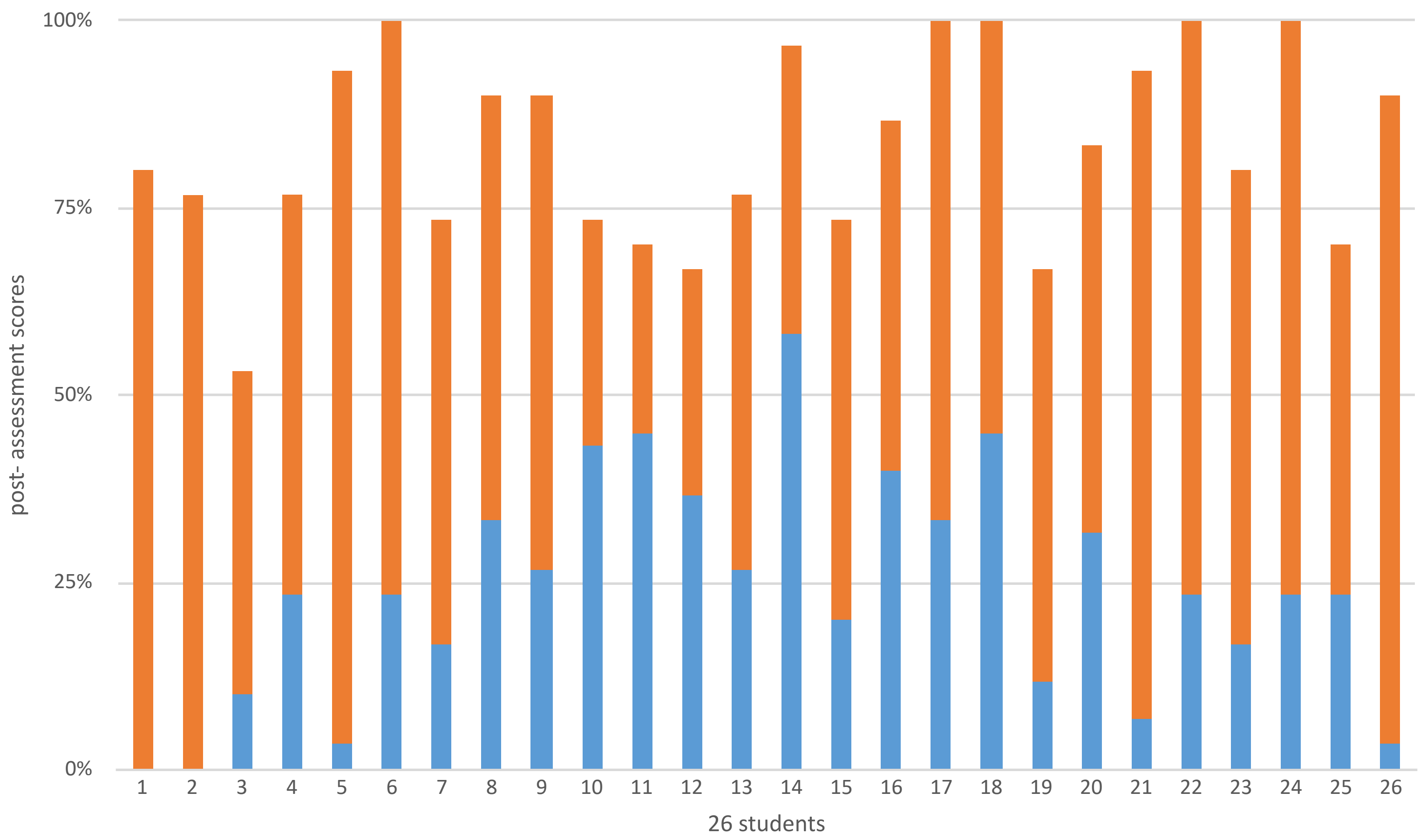
1. Address gaps in knowledge and skills that are not addressed by the current APMA 2120 Calculus III.
2. Provide an enriched experience to the advanced students that enhances their learning.
3. Use modern pedagogy with active learning in the classroom.

Pre-assessment and post-assessment to assess objective 1

Survey to assess objective 2 and 3

Pre-Assessment vs. Post-Assessment

A core set of questions: Newton’s Method, Integration Techniques, Simpson’s Rule, Applications of Integration, and Taylor series and Taylor polynomial applications.



Overall Approach to Teaching and Learning

Students were asked to rate how much different aspects of the class helped their learning
Engaging Student's Perspective (ESP) survey

1: no help	2: a little help	3: moderate help	4: much help	5: great help

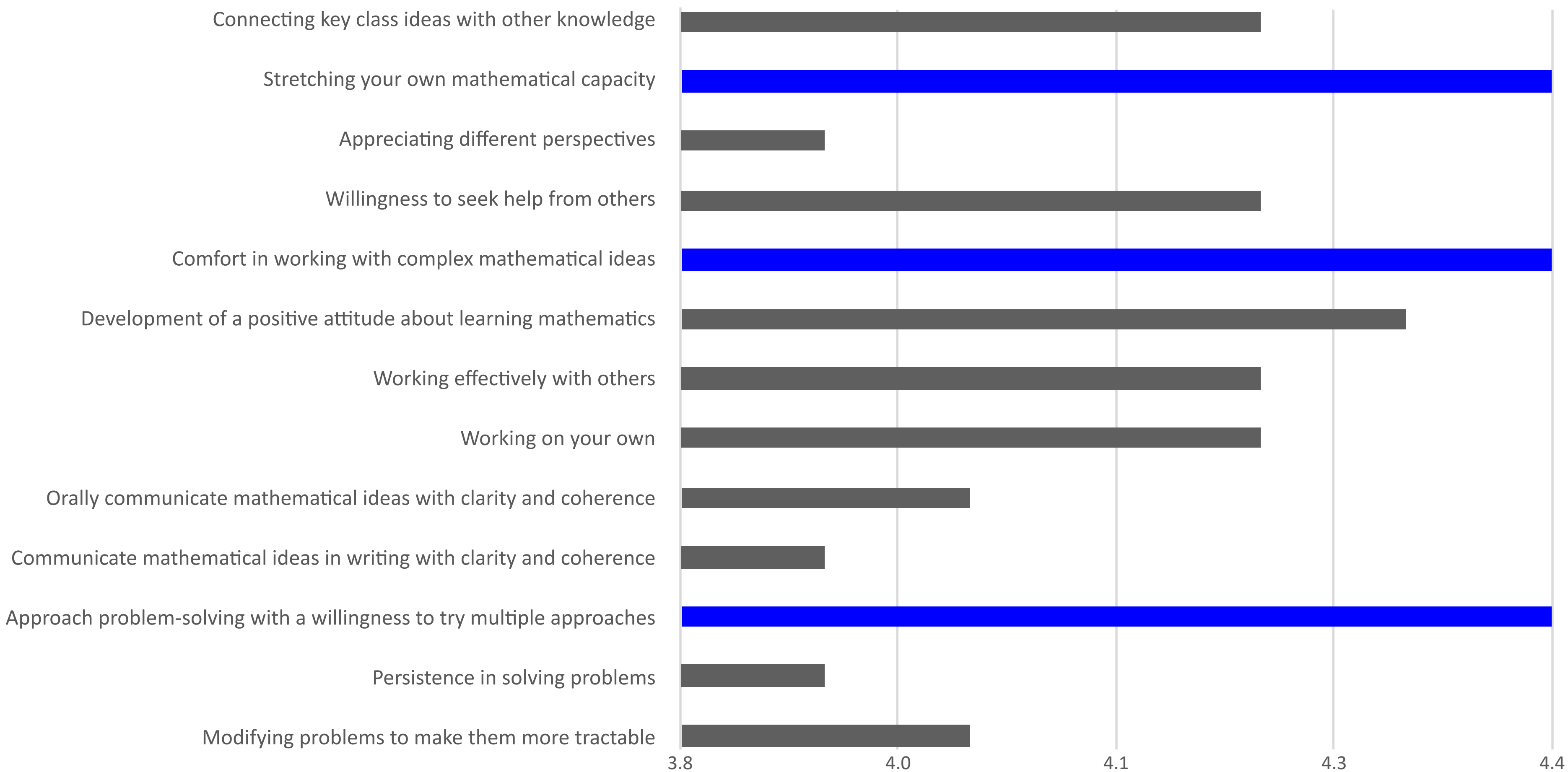
	Different Aspects of the Class Helped Student Learning	Mean
Class overall	The overall approach to teaching and learning in this course	4.5
	How class topics, activities, reading and assignments fit together	4.4
	The pace of the class	4.2
	The workload of the class	4.4
	The general atmosphere of the class	4.4
Class activities	Listening to mini lectures	4.1
	Participating in discussions during class	4.2
	Listening to discussions during class	4.1
	Participating in group work during class	4.2
	Explaining your work to other students	4.3
	Seeing other students explain their work	4.1
	Using MATLAB, Wolfram/Alpha or other technology when solving problems	2.6
Graded assignments	Doing reading quizzes before class	2.9
	Doing WebAssign Homework after class	4.5
	Group projects	3.0
	The feedback on your work received after tests	4.2
Class support	Interacting with the instructor during class	4.3
	Interacting with the instructor during office hours	4.4
	Working with peers during class	3.9
	Working with peers outside of class	4.0

Class Impact on Students' Skills and Attitudes

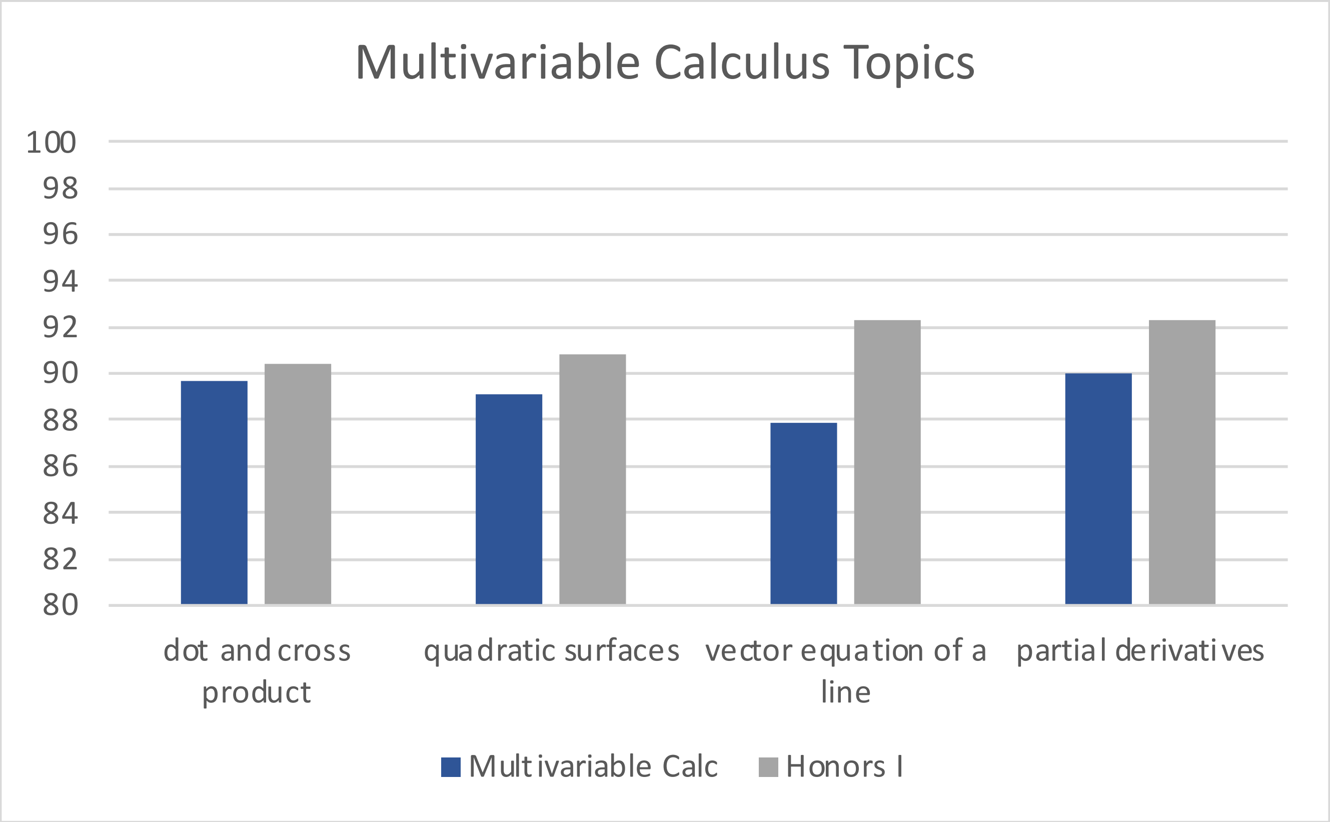
Student Assessment of Their Learning Gains (SALG) beginning and end of first semester

Students were asked to rate what gains they made in their skills and attitudes as a result of their work in this class.

1: no gains	2: a little gain	3: moderate gain	4: good gain	5: great gain

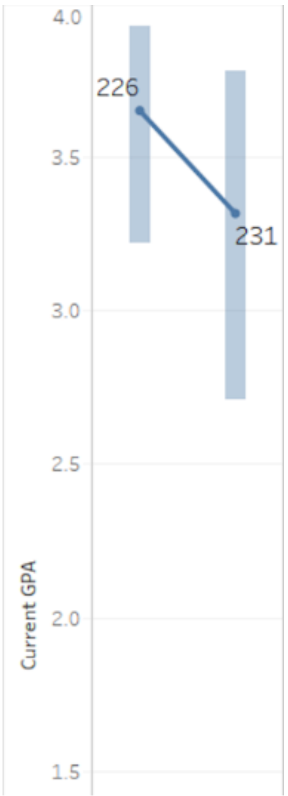


Honors vs Calc III

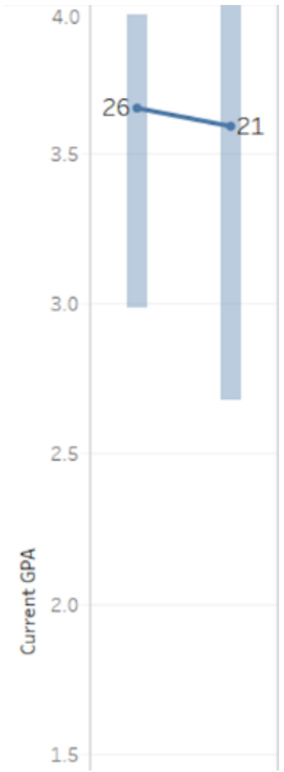


2nd Semester Dip

Calc II



Honors Eng Math



Our theory is the students' improved academic performance is due to their remaining together as a cohesive and highly functioning unit for both semesters.

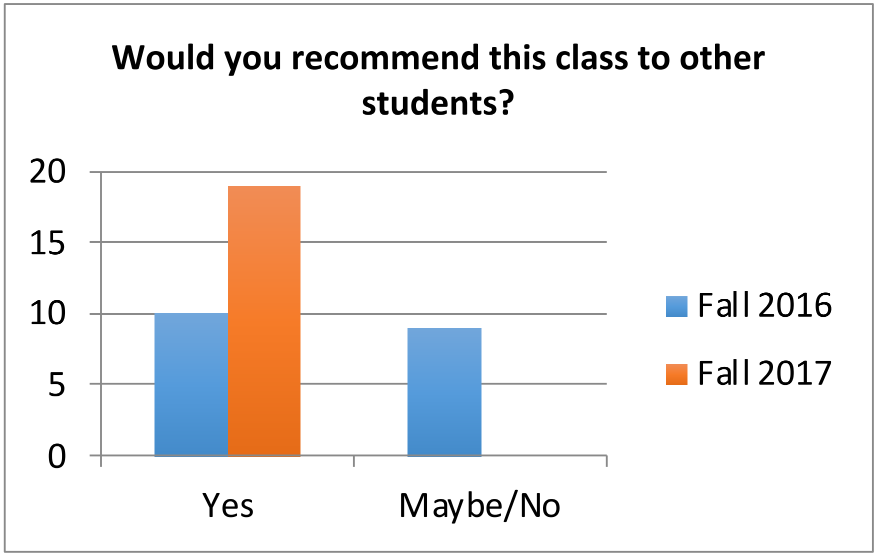
Student Comments

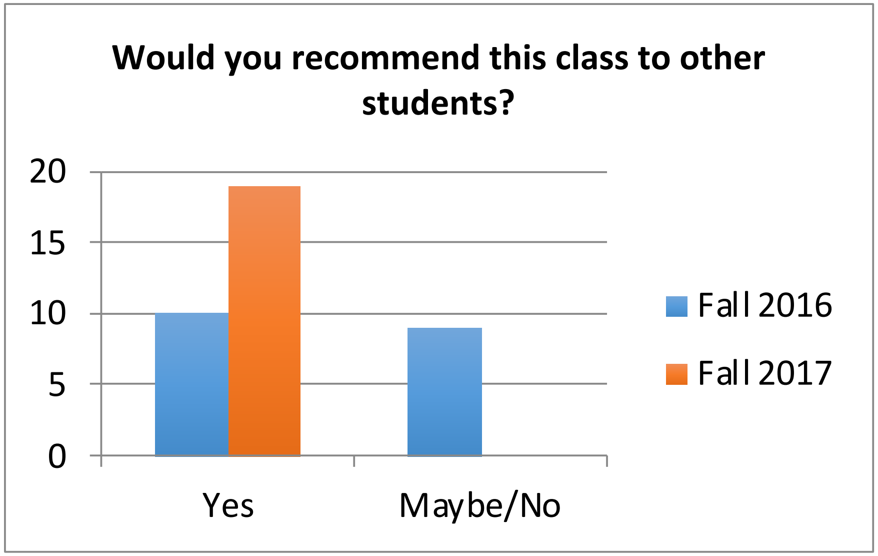
"I feel as though I have not participated as verbally in class discussions due to my quiet nature, but I have participated much more in this class than I have in others because of the welcoming environment. Additionally, because this class is very group-oriented, I feel I have gotten to know my classmates very well as a result. In fact, even outside the classroom, many of us meet up and discuss math and other topics."

Student Comments

"I'm glad I chose this course"
"I feel very confident in my decision"
"I am very thankful I took this course instead of multivariable"
"I am very glad I made that decision"
"I feel good about my decision. I think either way would have been about the same but I enjoy this class. "
"I don't regret it, though sometimes the 2-semester long aspect of it affected my courses in the second semester."
"I wish I had taken the traditional calculus course instead just because I do not like multivariable calculus and wish I had gotten it over wish sooner and I feel like this class took a lot of work"
"I kinda wish I would've got into Diff EQ faster, but I enjoyed many aspects of this class"
"Though sometimes I wish I had just taken multivariable calculus first semester to clear up class space, I feel as if the quality of learning I had in this course was worth it."

Student Comments & Improv





Improvements

- Group size for collaborative group work from four to three
- Adding a written homework
- Devoting a portion of class time every week for students to discuss group projects
- Employing more student presentations in class

Student Comments & Improv

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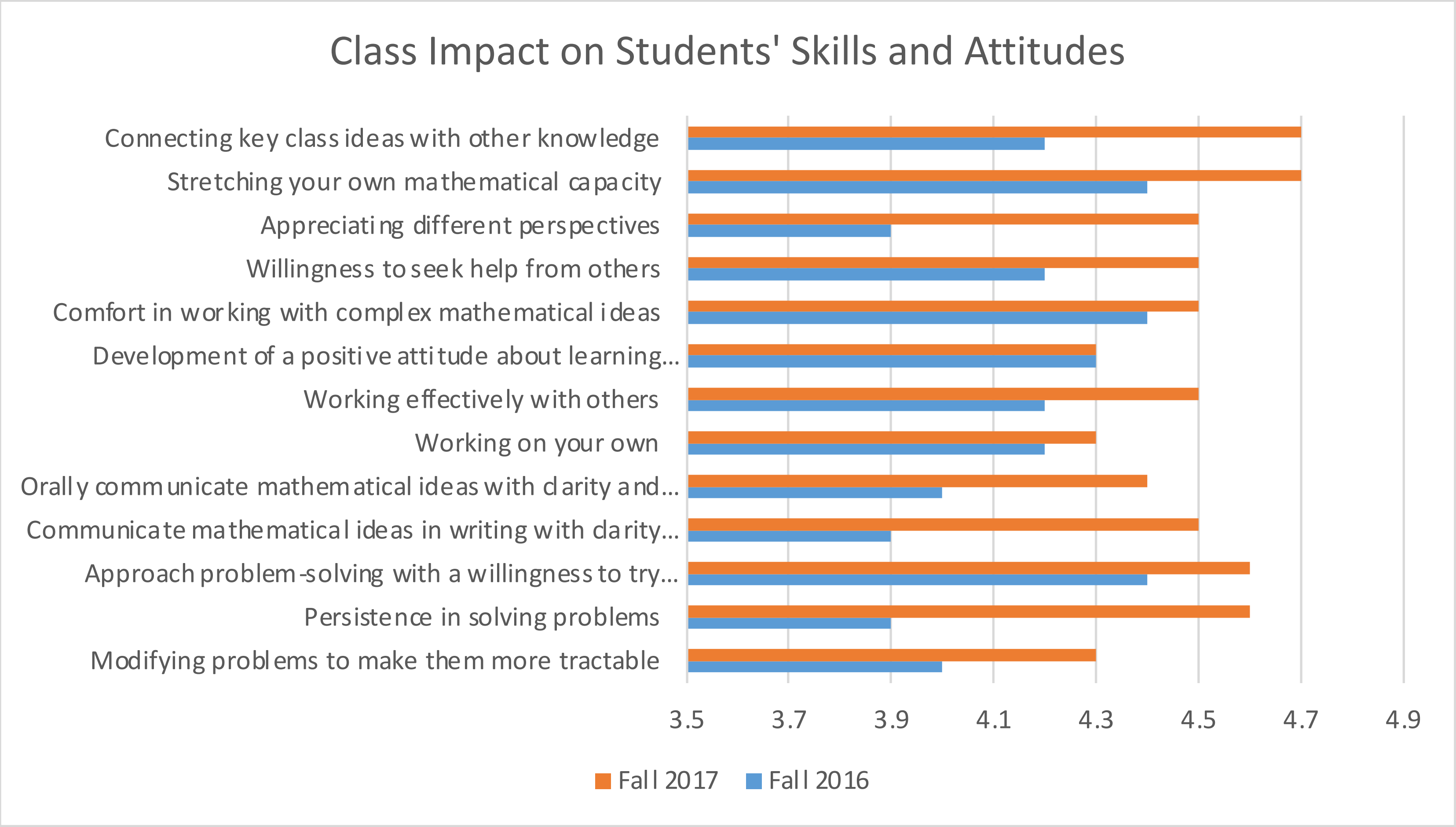

Engineering Math Project Rollout

EIA 2016

- Create Modules for Engineering Math Lab.
- Create *Essentials of Math for Engineers Assessment:*
  - Placement test to more properly place students in APMA.

Essentials of Math for Engineers Assessment

When students are empowered to make their own choices, how can they be persuaded to make a better choice?

Pick Calculus II even if not sufficiently prepared

1. Is free to students with no added direct costs to the institution.
2. Is fool-proof enough
4. Collects calculus background information like high school courses, grades, and AP test scores.
5. Assesses students' current abilities in Pre-Calc, Single Variable Calculus I, and Single Variable Calculus II.
6. Includes an algorithm to provide personalized placement recommendations
8. Includes a process to identify students who have improperly self-placed
9. Provides students with access to advisors regarding placement recommendations.
10. Tracks and links placement test results and recommendations to final exam results from the first semester to assess the quality of the recommendations.
11. Includes improvements to test questions and to the placement algorithm to improve matching in future years.

Essentials of Math for Engineers Assessment

Compliance rate was 74.1% (658 students)

60 questions the average score was 61.8%

Level	Number of Questions	Average Score (%)
Pre-Calculus	30	66.8
Single Variable Calculus I	15	70.5
Single Variable Calculus II	15	43.3

Pre-Calculus test questions were categorized as follows, with some questions included in more than one category:

Category	Number of Questions	Average Score (%)
Trigonometry	14	67.4
Logarithm/ Exponential Functions	10	61.5
Simplifying Expressions	5	63.2
Solving Equations	5	60.7
Limits	5	59.9
Graphing	5	80.1

Essentials of Math for Engineers Assessment

62 Calculus I students participated in the placement process

17 students had enrolled in Single Variable Calculus I before the placement test

39 students enrolled after the placement test but before the beginning of the semester, generally moving from Single Variable Calculus II.

6 more students switched from Single Variable Calculus II after the beginning of the semester.

In the past, migration of 2 to 3 dozen students from Single Variable Calculus II to Single Variable Calculus I after the beginning of the Fall semester.

Participating Students	Single Variable Calculus I	Single Variable Calculus II	Multivariable Calculus	HEM I
Initial Enrollment	17	176	148	19
Pre-Semester Enrollment	56	183	158	20
Final Enrollment	62	186	160	20

Essentials of Math for Engineers Assessment

Calculus I Results			
Placement	Final Exam z-score	GPA	Count
UNDER	0.024	4.00	1
AT	0.072	2.95	61
OVER			
No Placement Test	-0.219	2.85	24

Calculus II Results			
Placement	Final Exam z-score	GPA	Count
UNDER	0.021	3.67	3
AT	0.263	3.44	135
OVER	-0.386	2.58	48
No Placement Test	-0.368	2.83	55

Multivariable Calculus Results			
Placement	Final Exam z-score	GPA	Count
UNDER			
AT	0.228	3.45	146
OVER	-0.279	2.91	14
No Placement Test	0.082	2.76	65

Core Engineering Math

How do we cover the same material in less time?

- Our traditional Calculus I course begins with three weeks of pre-calculus review, but the Core sequence skips pre-calculus entirely. We recruited students for the Core track based on performance on a pre-calculus assessment.
- We also streamlined the coverage of integration techniques (for example, trigonometric substitution is omitted). The coverage of sequences and series was also adjusted. Now, applications and approximation methods are emphasized more than convergence tests.
- Our traditional calculus sequence covers parametric equations prior to sequences and series at the end of Calculus II. Then vectors are introduced at the beginning of Multivariable calculus. In the Core sequence, parametric equations are moved to the second semester and covered immediately before vectors, resulting in a more seamless transition.
- Finally, the traditional four-hour courses include a number of "discussion" sessions which allow for student questions. The Core sequence contains far fewer discussion sessions and expects students to attend office hours more frequently.

Core Engineering Math

From the Syllabus

The Core Engineering Math sequence targets a very specific first-year student. Some of our incoming students have a solid Calculus I background, but they aren't quite ready for Calculus II.

Students prepare for class by reading lecture notes (posted on UVaCollab).

During class, students worked in self-selected groups of two or three.

Typically, the first few worksheet exercises were very simple, primarily focused on motivating the new material. Gradually, the exercises built in complexity until students were tackling the most difficult facets of the topic in question.

At the end of class (or the beginning of the next class for a longer assignment), each student submitted the assignment for grading.

- **Homework (10%)**
- **Check-for-Understanding (10%)**
- **3 Midterms (45%)**
- **Final Exam (25%)**

Core Engineering Math I

Final Exam 2501

Section	Topic	Average	Proficiency
2	Limits and Derivatives	62.5	Fair
3	Differentiation Rules	82.1	Good
4	Applications of Differentiation	81.8	Good
5	Integrals	86.7	Good
6	Applications of Integration	73.3	Fair
7	Techniques of Integration	68.7	Fair
11	Infinite Sequences and Series	57.8	Poor

Many students struggled with sequences and series one of the most difficult topics in the course.

Core Engineering Math I

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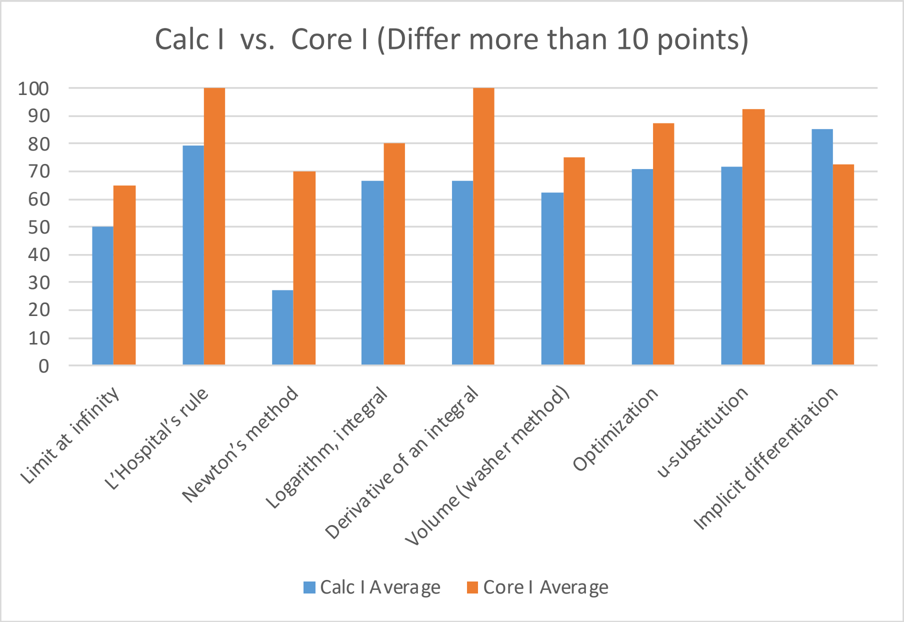
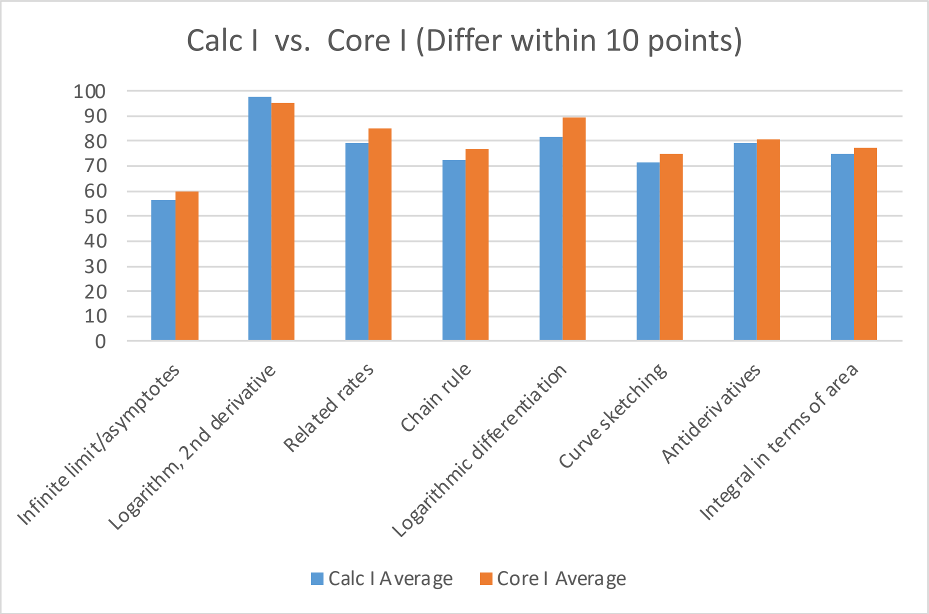
APMA 2501	APMA 1090			
Average	Average	Diff	Topic	Section
60	56.3	3.7	infinite limit	2.2
65	50	15	limit at infinity	2.6
76.7	72.6	4.1	chain rule	3.4
72.5	85.1	-12.6	implicit differentiation	3.5
95	97.9	-2.9	2nd derivative of log	3.6
89.2	81.6	7.6	logarithmic differentiation	3.6
85	79.2	5.8	related rates	3.9
75	71.4	3.6	curve sketching	4.3
100	79.2	20.8	l'Hospital's rule	4.4
87.5	70.8	16.7	optimization	4.7
70	27.1	42.9	Newton's method	4.8
80.8	79.2	1.6	antiderivative	4.9
77.5	74.7	2.8	integral as area below curve	5.2
80	66.7	13.3	definite integral (exp)	5.3
100	66.7	33.3	derivative of integral	5.3
92.5	71.5	21	u-substitution	5.5
75	62.5	12.5	solid of revolution	6.2
50	85.4	-35.4	Taylor poly (approx of root)	11.11

Many students struggled with sequences and series one of the most difficult topics in the course.

Core Engineering Math I





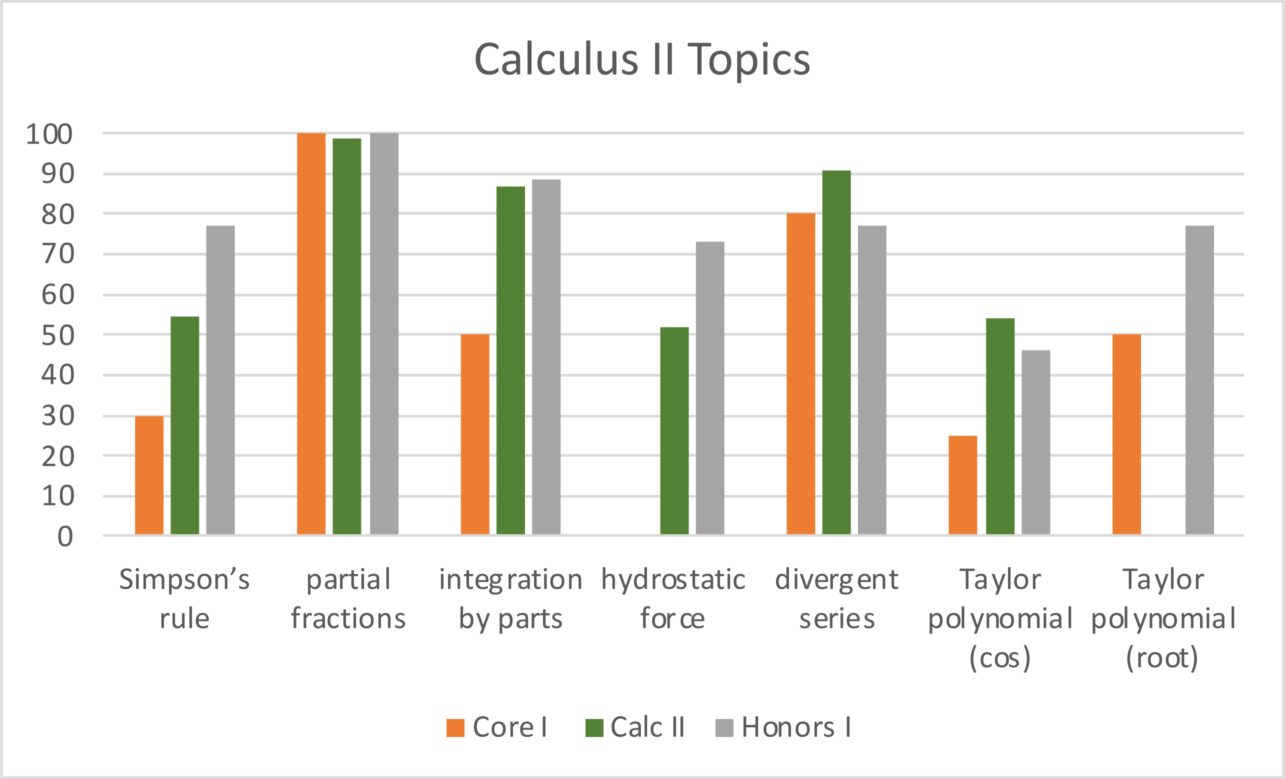
The traditional Calculus I course covers less material at a slower rate, but the students are less able to master the material when compared to students in the Core track.

In comparison, the first semester of the Core track covers all of the Calculus I material, plus integration techniques, some applications of integration, and sequences and series

Core Engineering Math I

Topic Covered:	Calc I	Calc II	MV Calc	Core I	Honors I
Newton's method	27.1			70.0	92.3
Simpson's rule		54.4		30.0	76.9
partial fractions		98.9		100.0	100.0
integration by parts		86.8		50.0	88.5
hydrostatic force		52.1			73.1
divergent series		90.6		80.0	76.9
Taylor polynomial (cos)		53.9		25.0	46.2
Taylor polynomial (root)				50.0	76.9
dot and cross product			89.7		90.4
quadratic surfaces			89.1		90.8
vector equation of a line			87.9		92.3
partial derivatives			90		92.3

Core Engineering Math I



Core Engineering Math II

Final Exam 2502

Section	Topic	Average	Proficiency
8	Further Applications of Integration	59.9	Poor
10	Parametric Equations & Polar Coordinates	86.1	Good
12	Vectors and the Geometry of Space	62.6	Fair
14	Partial Derivatives	71.3	Fair
15	Multiple Integrals	78.4	Good
16	Vector Calculus	69.2	Fair

Core E

Final Exam 2502

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8	Further Applications of Integration	59.9	Poor
10	Parametric Equations & Polar Coordinates	86.1	Good
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14	Partial Derivatives	71.3	Fair
15	Multiple Integrals	78.4	Good
16	Vector Calculus	69.2	Fair

APMA2502 Average	APMA2120 Average	Diff	Topic	Section
94.4	77.8	16.6	Dot&Cross_Products	12.3
94.4	100	-5.6	Cross_Product	12.4
33.3	61.1	-27.8	Lines&Planes	12.5
65	63.9	1.1	Equation_of_a_Plane	12.5
72.2	77.8	-5.6	Continuity&Partial_derivatives	14.2
94.4	100	-5.6	Partial_Derivatives	14.3
56.1	81.7	-25.6	Tangent_Plane	14.4
33.3	83.3	-50	Directional_Derivative	14.6
61.1	72.2	-11.1	Directional_Derivative	14.6
79.2	83.9	-4.7	Directional_Derivative&Gradient	14.6
80	79.4	0.6	2nd_Derivative_Test	14.7
83.3	100	-16.7	Double_Integral&Symmetry	15.1
76.4	99.4	-23	Center_of_Mass	15.4
78.5	90	-11.5	Volume&Spherical_Coordinates	15.8
55.6	66.7	-11.1	Vector_Line_Integrals	16.2
44.4	77.8	-33.4	Fundamental_Thm_of_Line_Integrals	16.3
78.4	84.4	-6	Fundamental_Thm_of_Line_Integrals	16.3
55.6	55.6	0	Green's_Thm&Area	16.4
70.6	80.6	-10	Green's_Thm	16.4
88.9	88.9	0	Curl	16.5
77.8	94.4	-16.6	Divergence	16.5
88.9	100	-11.1	Curl_of_a_Conservative_VF	16.5
72.2	94.4	-22.2	Parametric_Surfaces	16.6
61.1	50	11.1	Surface_Integrals	16.7
61.7	67.2	-5.5	Stokes'_Thm	16.8
72.8	84.4	-11.6	Divergence_Thm	16.9

Core Engineering Math II

- APMA 2120 (Calc III) students performed better on the common problems.
- In addition to the multi-variable topics (chapters 12-16), they also covered the second half of single variable integral calculus (chapters 8 and 10).
- About half of the students in APMA 2502 (Core Eng Math II) excelled in the course despite the additional course material.
- Many students in 2502 (Core Eng Math II) clearly struggled with the pace of the course, resulting in significantly lower scores on the final exam.
- For the vast majority of students, two semesters is not sufficient to effectively cover all of single and multi-variable calculus.

Bernard's feedback:

I don't think that the Core Engineering Math sequence should be offered in Fall 2018.

Core - Honor Engineering Math

- Honors track students are best prepared in general, and tended to perform better than the other tracks for the common problems in all areas.
- Core track students are generally better prepared than the traditional Calculus I students and performed better for the Calculus I topics.
- In general the preparation of the Calculus II students lies between the Core track and the Honors track and their performance on the Calculus II topics tended to lie between as well.

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Core Engineering Math Track was not offered in 2018

Honors Engineering Math Track ended with Covid (Spring 2020)

Conclusions?

What is missing?

Gender and Demographic
First Generation
Under Represented Minorities

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Exposure to math?

Conclusions?

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Exposure to math?

Inability to process basic content information

$$\sqrt{9+9} = \sqrt{9} + \sqrt{9}$$

$$\frac{e^{2x} - e^{-2x}}{4} = \left(\frac{e^{x} - e^{-x}}{2}\right)^2$$

$$\frac{1}{\frac{2}{x} + \frac{3}{x^2}} = \frac{x}{2} + \frac{x^2}{3}$$

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Students' fragility?

Conclusions?

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Students' fragility?

A great instructor, and a great course, cannot fix a structural problem

Funding

UVA Center for Teaching Excellence (former Teaching Resource Center)

Nucleus Grant 2015-2016

UVA School of Engineering

2016-2017 Education Innovation Awards

2017-2018 Education Innovation Awards

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Thanks!