Calculus at Multiple Scales: Successes and Challenges

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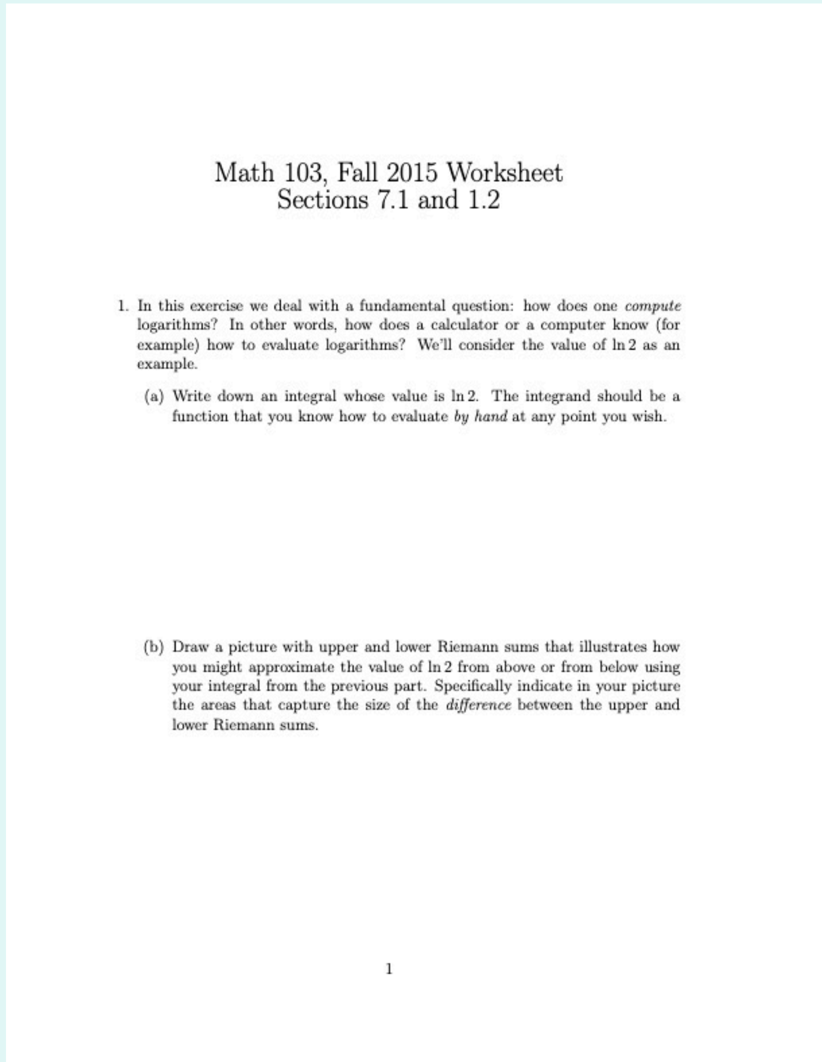
Many instructors have faced the challenge of adapting classroom practices from one setting to another. Like houseplants, some approaches that thrive in one setting seem to mysteriously wither in others. In this talk, I will focus on the effect of scale in (mostly) my own experience teaching single variable integral calculus. In the pre-pandemic era, this includes a five-year run teaching an uncoordinated active calculus section of 50-60 students which eventually produced reasonable successes building student buy-in and promoting social belonging (the "local" scale). During the pandemic, the course evolved into an online, coordinated version with sections of 120 (the "institutional" scale) and has continued to grow in this direction now that in-person instruction has resumed. Among the unsolved challenges in this new format are several practical and technological issues in particular that I would like to highlight for comparison and discussion across institutions and contexts (the "global" scale).

The Local Scale

My Initiation

- Single variable integral calculus (mostly); class sizes of 50-60
- Dedicated instructional space (10 round tables seating 6 each, 360° whiteboards, no clear "front" of room)

- FTP repository of worksheets donated by a previous instructors, which were my first real encounter with worksheets at this level.



Math 103, Fall 2015 Worksheet
Sections 7.1 and 1.2

1. In this exercise we deal with a fundamental question: how does one *compute* logarithms? In other words, how does a calculator or a computer know (for example) how to evaluate logarithms? We'll consider the value of $\ln 2$ as an example.

   (a) Write down an integral whose value is $\ln 2$. The integrand should be a function that you know how to evaluate *by hand* at any point you wish.

   (b) Draw a picture with upper and lower Riemann sums that illustrates how you might approximate the value of $\ln 2$ from above or from below using your integral from the previous part. Specifically indicate in your picture the areas that capture the size of the *difference* between the upper and lower Riemann sums.

1

The First Reactions

- I was surprised and confused by how much the students struggled with the worksheets.
- Students strongly disliked the class: "overall quality of the course" was rated at 1.04 / 4.0 with the average of all sections (mine + 2 others) being 1.97 / 4.0.

> *I would take another class from professor Gressman only if it was not a SAIL class. The SAIL class did not allow Professor Gressman, who I'm sure is a great teacher, to teach whatsoever. Throughout the entire semester, we did not once have an in-class lecture, or do anything in class that was not simply completely an 8 page packet on material that nobody was completely sure about or comfortable with. SAIL method also, in my opinion, made the class much more difficult. After speaking with others who were in the normal lecture Math 103 class, they seemed much more confident going into midterms and doing work than most people in this class. It was very easy to get off task and not get the required help when doing in class work. Quality of instructor was fine, but the quality of the class itself was very, very poor. If I had known the quality of this course prior to taking it, I would not have spent the money to sit in a classroom, for 1.5 hours twice a week, to complete a packet every day.*

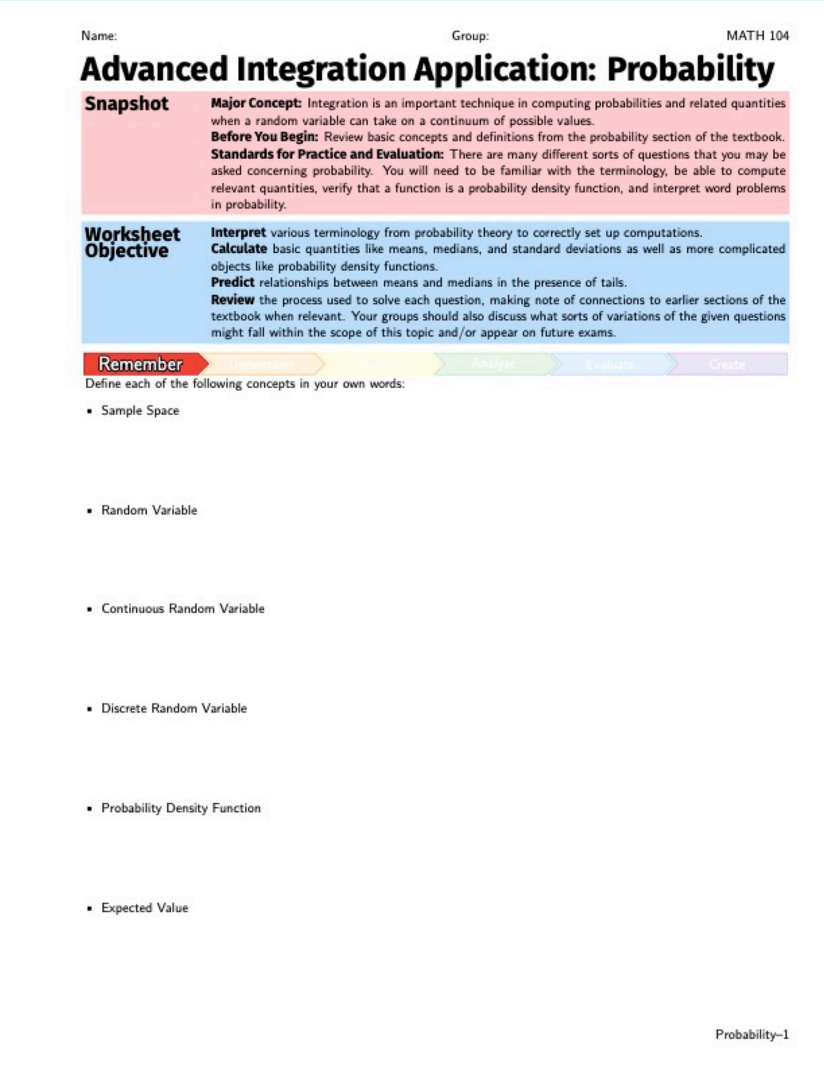
Correcting Course

- I joined the SAIL teaching community at Penn through our Center for Teaching and Learning. This quickly highlighted basic best practices and avoidable errors.
  - Group size/composition/shuffling
  - Watching for personality dynamics within groups
  - Discouraging "divide and conquer" approach by students
- I recognized that my own reaction to seeing students struggle was prompted by the fact that I had never actually witnessed this part of the learning process before.
- I recognized that I would need to listen to, face, and eventually anticipate some of the tougher student questions / concerns (e.g., "Why do we need to learn this?")
- Developed Priorities for Improvement
  - **Radical Transparency** both in modeling thinking and in scaffolding worksheets
  - **Explicitly Building Buy-In** by devoting small amounts of class time to discussion of various structural choices
  - **Implicitly Building Buy-In** by making the materials look "high quality" and emphasizing that they are something to be kept
  - **Promoting a Sense of Social Belonging** to counteract strong social and psychological forces which tax students unequally

Examples

Transparency and Implicit Buy-In Examples

Worksheet Example



Name: Group: MATH 104

Advanced Integration Application: Probability

Snapshot

Major Concept: Integration is an important technique in computing probabilities and related quantities when a random variable can take on a continuum of possible values.
Before You Begin: Review basic concepts and definitions from the probability section of the textbook.
Standards for Practice and Evaluation: There are many different sorts of questions that you may be asked concerning probability. You will need to be familiar with the terminology, be able to compute relevant quantities, verify that a function is a probability density function, and interpret word problems in probability.

Worksheet Objective

Interpret various terminology from probability theory to correctly set up computations.
Calculate basic quantities like means, medians, and standard deviations as well as more complicated objects like probability density functions.
Predict relationships between means and medians in the presence of tails.
Review the process used to solve each question, making note of connections to earlier sections of the textbook when relevant. Your groups should also discuss what sorts of variations of the given questions might fall within the scope of this topic and/or appear on future exams.

Remember

Define each of the following concepts in your own words:

- Sample Space
- Random Variable
- Continuous Random Variable
- Discrete Random Variable
- Probability Density Function
- Expected Value

Probability–1

Explicit Buy-In Examples

- Second week of class: brief conversation about Freeman *et al*., focusing on interpreting the graphs, summarizing findings, and explaining the practice as it appears in our classroom
- Fifth week of class: conversation about Deslauriers, McCarty, Miller, Callaghan, and Kestin "*Measuring actual learning versus feeling of learning in response to being actively engaged in the classroom*" (active versus traditional instruction in Harvard physics class)

Social Belonging

Prompted by the book "*Whistling Vivaldi: How Stereotypes Affect Us and What We Can Do*" by Claude Steele and based on an exercise described by Aguilar, Walton, and Wieman.

Early in the semester, (2-3 weeks before first midterm) show students thoughts and comments addressed to them by students from previous classes.

Near the end of the term, ask students to prepare advice for future generations of students:

> Many new students in Math 104 find themselves facing anxiety about things like their previous academic preparation, difficulty feeling like they belong in the course, shifting understandings of grades and academic standards, or concerns about keeping up in such a fast-paced course. As time goes on, most find that such anxieties are far more common among peers than it initially appeared and that there is a natural adjustment period during which they come to feel more capable and confident in their own ability to succeed.
>
> The purpose of this assignment is to use your unique perspectives as recently new students to generate meaningful, personal insights that can be compiled and shared as a valuable resource for next fall's incoming Math 104 students.
>
> Reflect your own personal experience in Math 104 so far and write about the ways that you see it illustrating the general processes of intellectual growth, learning to understand and manage obstacles and anxieties, and gaining confidence in your own abilities.
>
> You should aim to keep responses relatively short (1-2 paragraphs maximum). Submissions will be graded anonymously in Canvas. All submissions received before the deadline will receive full credit.

Results

- The grade distribution became much more compact—no more tail of students doing vastly worse than their peers
- Students actually "liked" the class: "overall quality of the course" was rated at 2.55 / 4.0 with the average of all sections being 2.50 / 4.0.

> *[...]The active learning classroom was a great way to learn, much better than a lecture style course[...]*

> *I really enjoyed being in an interactive class as it forced me to become active with my learning by learning the content before coming to class and asking questions in class.*

The Institutional Scale

Understanding the Populations

Calculus instruction at the University of Pennsylvania looks like this:

Students

- **Math 103:** Single variable differential calculus
  - Approximately 300 students per year
  - About 50% continue to Math 104
- **Math 104:** Single variable integral calculus and applications; sequences and series
  - Approximately 1,000 students per year
  - Recommended starting course for successful completion of Calculus AB
  - Approximately 30-40% continue to Math 114
- **Math 114:** Multivariable calculus
  - Approximately 1,000 students per year
  - Recommended starting course for students with 5 on Calculus BC exam

All incoming students take an automated diagnostic exam via LMS to get a non-binding placement recommendation.

Instructors

- **Math 103:** Approximately 2 instructors in fall, 1 in spring
- **Math 104:** Approximately 7 instructors in fall, 3 in spring
- **Math 114:** Approximately 6 instructors in fall, 4 in spring

Instructors are:

- 60% Postdocs and Lecturers (most are early career or even first-time instructors; 3-year positions)
- 25% Senior Lecturers
- 15% Tenure track and Tenured Faculty
- Median instructor teaches calculus 3 times during their employment here

The Format of Calculus at Penn

- **Prerecorded Video** (Is it a book or a lecture?)
  - Approximately 60 minutes per week of assigned video content per week.
  - Students describe videos as "conceptual."
  - Accompanied by quizzes in LMS which ask students to do some thinking beyond the recall/understand stages.
- **Instructor Contact**
  - Instructors spend one 90-minute session per week with each student.
  - Sections of 120 are split into halves and meet with instructor on alternating days (M and W or T and R).
  - Class time emphasizes active learning.
- **TA Contact**
  - TAs meet with students for one hour per week in groups of 30 on days they are not meeting with the instructor.
  - Format of recitation varies; can be worksheet-based or take the form of a problem session.

- **Friday Quizzes**
    - Approximately 12 weekly quizzes per semester
    - Divided into batches of 4; first three cover material from current and prior week; fourth quiz is cumulative over the last month
    - Students automatically drop the lowest quiz score in each batch of 4
    - Each quiz is approximately 4 questions at 10 minutes/question.
    - Each quiz is worth about 5% of total grade
    - There are no midterm exams, and the weight of the final exam is diminished

What's Working Well

- Students prefer quizzes to midterms and appreciate being able to drop several low scores.
- Instructors report positive effects of frequent assessment–students keep up with material and achieve higher scores on assessments.
- In my own online section in Fall 2020, I managed to find (after some casting around) a reasonably effective way to port worksheets online.

Challenges

- Achieving *student* buy-in around the nontraditional format and materials is difficult.
- Achieving *instructor* buy-in around the nontraditional format is difficult.
  - First-time teachers greatly appreciate high coordination and availability of teaching materials, but often would prefer a more vanilla format and approach.
  - Tenured faculty feel constrained by high coordination.
- Some instructors and students would prefer more frequent contact.
- Logistical challenges mix ordering of lecture and recitation inside the week.

Some Ideas Out There

- Giving students clearer signals about how to feel about the video content and what specific understanding is necessary for the upcoming lecture.
- Stronger messaging to students ***and instructors*** about unique features of structure and approach.
- More substantial training for first-time instructors.
- Swapping 90 minutes 1x/wk for each half with 45 min 2x/wk with the whole section.
- More / all grading on mastery basis? (Students like the curve when they feel it benefits them.)

The Global Scale

We were delighted to ditch the publisher. I am not sure that I fully appreciated that we would ***become*** the publisher.

Content Creation

- Content creation is time-consuming: both invention and presentation are hard.

"One-Time" Work

- Virtual Texts: Differential and Integral Calculus authored in PreTeXt with embedded WeBWorK and interactive elements
- Video Series

- Big pushes by small groups of dedicated individuals
- Digital Decay: One-time work unfortunately does not live up to its name. LMS-based content is not future-proofed on the LMS side and degrades over time as the LMS software "improves."

- This is the time-consuming, everyday work for large numbers of instructors
- Lots out there: AMS Open Math Notes, Cornell Good Questions Project, opencalculus, calculus.org, MIT Open Courseware, Knewton Alta, Edfinity, DigitalEd, Derivita, Gradarius. These provide different things and have different degrees of portability into existing courses.
- There are really two parts (at least) to the writing of computational problems: the underlying computation to be done and the setting/presentation.
  - In calculus, there is an essentially finite list of underlying computations that can reasonably be done by hand.

44. $\boxed{\textbf{Arclen44}}$
Compute the arc length of the curve

$$y = -\frac{2}{3}x^{3/2} + 1 + \frac{1}{2}x^{1/2}$$

between the endpoints $x = \frac{1}{4}$ and $x = 1$.

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ **(c*)** $\frac{5}{6}$ (d) 1 (e) $\frac{7}{6}$ (f) $\frac{4}{3}$

Feedback: Applying the formula for arc length gives that

$$\begin{aligned} L &= \int_{\frac{1}{4}}^{1} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \int_{\frac{1}{4}}^{1} \sqrt{1 + \left(-x^{1/2} + \frac{1}{4}x^{-1/2}\right)^2}\, dx \\ &= \int_{\frac{1}{4}}^{1} \sqrt{1 + x^1 - \frac{1}{2} + \frac{1}{16}x^{-1}}\, dx = \int_{\frac{1}{4}}^{1} \sqrt{\left(x^{1/2} + \frac{1}{4}x^{-1/2}\right)^2}\, dx \\ &= \int_{\frac{1}{4}}^{1} \left(x^{1/2} + \frac{1}{4}x^{-1/2}\right)\, dx = \left(\frac{2}{3}x^{3/2} + \frac{1}{2}x^{1/2}\right)\Big|_{\frac{1}{4}}^{1} \\ &= \left(\frac{2}{3} + \frac{1}{2}\right) - \left(\frac{1}{12} + \frac{1}{4}\right) = \frac{5}{6}. \end{aligned}$$

Note that you must always take the positive square root in going from line two to line three. In particular, if you get a negative answer, you have likely taken the negative square root.

  - Different settings / variations are a significant source of challenge for most students.

Call for Aid

- Instructors have increasingly many platforms for distributing mathematical content (physical materials, electronic pdfs, home-brew websites + MathJax, PreTeXt, Ximera, LMS Quizzes) and sometimes limited autonomy over delivery options. LMSs in particular strip away significant control of layout/formatting. Each destination has its own format requirements.
- Getting content *out of* sites like LMSs is difficult and cross-platform portability is almost nonexistent.
- There is great need for storing content in stable, shareable, format-agnostic formats.
- TeX/LaTeX make choices that decrease human readability but are nevertheless the dominant language for transmitting mathematical expressions.
- I believe that we would benefit as a community by developing a **standardized format for the development and distribution of instructional materials.**
- Content should be designed for maximum flexibility. I believe this means decoupling
    - computational details,
    - question type/context, and
    - delivery format/intended use
- Authoring flexible underlying content needs to be **extremely simple**. It's okay (desirable?) if it's hard to write highly rigid content.
- Most of the heavy lifting needed to make student-ready materials should be automated.

Playing with Ideas

Randomly-selected item from Good Questions Project, LaTeX source:

```
\item {\bf [Q]} Newton's method is a cool technique, because:
\begin{enumerate}
\item It can help us get decimal representations of numbers like
$\sqrt[4]{3}$, $\sqrt[8]{5}$ and $\sqrt[5]{13}$
\item It can be used to find a solution to $x^7=3x^3+1$
\item Both (a) and (b).
\end{enumerate}

{\it Answer: (c).} Quick check of when we use Newton's method.
```

Same question, typed in markdown-like syntax:

```
QUESTION: **[[Q]]** Newton's method is a cool technique, because:
    a. It can help us get decimal representations of numbers
        like $\sqrt[4]{3}$, $\sqrt[8]{5}$ and $\sqrt[5]{13}$
    b. It can be used to find a solution to $x^7=3x^3+1$
    c. Both (a) and (b).

    ANSWER: (c). Quick check of when we use Newton's method.
```

Rendered Question:

Question

[Q] Newton's method is a cool technique, because:

a. It can help us get decimal representations of numbers like $\sqrt[4]{3}$, $\sqrt[8]{5}$ and $\sqrt[5]{13}$
b. It can be used to find a solution to $x^7 = 3x^3 + 1$
c. Both (a) and (b).

▶ Answer

List Semantics

```
+ Lorem ipsum dolor sit amet, consectetur adipiscing elit.
    Suspendisse iaculis semper sem, vel posuere purus
    sollicitudin nec. Cras sit amet ipsum malesuada
    lorem ullamcorper tempor eu ut arcu. Suspendisse
    elementum eleifend lacus, et aliquet mauris aliquam luctus.
+ Orci varius natoque penatibus et magnis dis parturient
    montes, nascetur ridiculus mus. Nulla consectetur augue
    ante, quis venenatis dui faucibus ac. Mauris accumsan
    dignissim justo et ullamcorper. In aliquet blandit
    eleifend. Praesent vulputate quam nisl.
```

Rendering 1:

- Lorem ipsum dolor sit amet, consectetur adipiscing elit. Suspendisse iaculis semper sem, vel posuere purus sollicitudin nec. Cras sit amet ipsum malesuada lorem ullamcorper tempor eu ut arcu. Suspendisse elementum eleifend lacus, et aliquet mauris aliquam luctus.
- Orci varius natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Nulla consectetur augue ante, quis venenatis dui faucibus ac. Mauris accumsan dignissim justo et ullamcorper. In aliquet blandit eleifend. Praesent vulputate quam nisl.

Rendering 2:

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Suspendisse iaculis semper sem, vel posuere purus sollicitudin nec. Cras sit amet ipsum malesuada lorem ullamcorper tempor eu ut arcu. Suspendisse elementum eleifend lacus, et aliquet mauris aliquam luctus.

Orci varius natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Nulla consectetur augue ante, quis venenatis dui faucibus ac. Mauris accumsan dignissim justo et ullamcorper. In aliquet blandit eleifend. Praesent vulputate quam nisl.