

# Onboarding Instructors in an Active Linear Algebra Course

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University of Toronto

# Dedication



Alfonso Gracia-Saz

1976–2021

Educator, Mentor, Friend

Post your memories of Alfonso at

[https:](https://uoft.me/alfonso-memorial)

[//uoft.me/alfonso-memorial](https://uoft.me/alfonso-memorial)

# The Course

Linear Algebra I, MAT223 (*per semester*)

- ~1100 students
- ~7 sections
- ~7 instructors (mostly inexperienced)
- First-year students (mostly inexperienced)

# The Project (c. 2018)

## Redesign Linear Algebra I, MAT223

- Revamp curriculum
- Provide a uniform student experience
- Introduce active-learning teaching strategies in every section

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- Provide training and support.
- Lower the barrier-of-entry to active learning.
  - Provide high-quality materials and guides.

# What does active learning look like in MAT223?

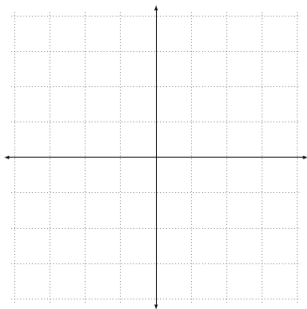
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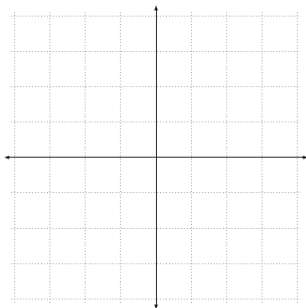
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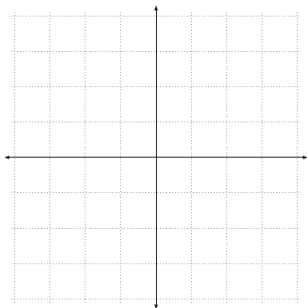
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- 2 Individual think time

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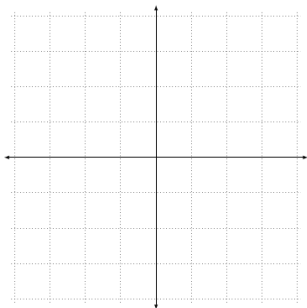
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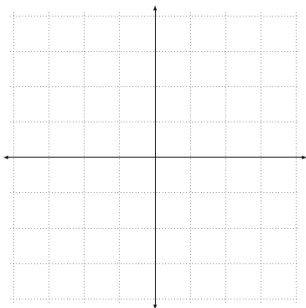
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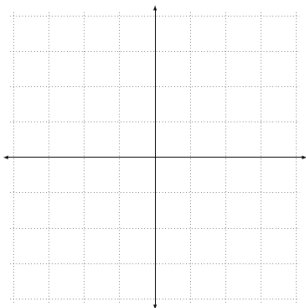
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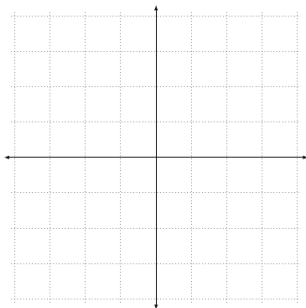
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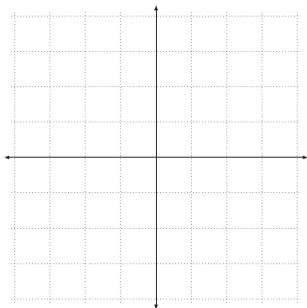
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What are some prerequisites/challenges in teaching this way?

Share your thoughts at:

<https://tinyurl.com/ACTIVE-ONBOARDING>

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- Created instructor guide to in-class exercises.

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## The Instructor Guide

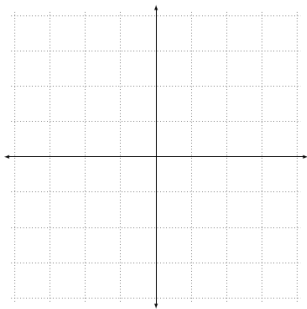
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The goal of this problem is to

- Visually see how the set of all vectors orthogonal to a given vector forms a line.
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### Notes/Misconceptions

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Example instructor margin notes:

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Support provided:

- Open invitations to visit my lecture.
- Teaching observations.

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- One instructor “gave up” and lectured directly from the book.
- 2/3 of instructors *didn't* adopt an active-learning style.

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**It's as if four hours of training are not enough!**

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- Instructors try, are *interrupted*, and retry.

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## Step \*\*: TA Support

- Lecture TAs for every day of class
- Lecture TAs are trained in both **how to help students** and **how to help instructors**.

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**Thank You**



# Instructor Difficulties

## Role Playing *Common Issues*

Instructors tend to...

- Give hints on a problem before students start.
- Explain details without measuring student's knowledge.
- Offer their own explanations rather than build off student knowledge.

All of these issues come up and are discussed.

## Classroom *Common Issues*

- Everything from role playing, and
- Suggestions rather than instructions (e.g., “if you'd like, discuss with your group”).
- Having “office hour”-style discussions instead of whole-class discussions.
- Transitions, transitions, transitions!