Ask. Don't Tell. – Annotated Examples

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The three principles:

1. Where is the student?
2. Help minimally.
3. Ask. Don't Tell.

1 Brett

Brett writes on piazza

> Derivatives proof through limit. Is it right if I say:
>
> $$f(a+h) = f(a) + h$$
>
> Shouldn't it be:
>
> $$f(a+h) = f(a) + f(h)$$

Your tasks:

- What is Brett really asking?
- How do you help Brett?

See next page for what happened

What actually happened

Brett writes on piazza

> Derivatives proof through limit. Is it right if I say:
>
> $$f(a+h) = f(a) + h$$
>
> Shouldn't it be:
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> $$f(a+h) = f(a) + f(h)$$

Instructor:

> $f(a+h)$, $f(a)+f(h)$, and $f(a)+h$ are three completely unrelated things. None of them is necessarily equal to any other one.
>
> Exercise: Come up with functions that demonstrate this. Seriously. Do it, and give your answer below.

Brett:

> I see,
>
> $f(x) = 2x, a = 2, h = 4$
>
> $f(a+h) = f(6) = 12$
>
> $f(a) + f(h) = f(2) + f(4) = 4 + 8 = 12$
>
> $f(a) + h = f(2) + 4 = 4 + 4 = 8$

Instructor:

> Now, give an example where all three are different.

Brett:

> $f(x) = 2x + 1, a = 2, h = 4$
> $f(a+h) = f(6) = 13$
> $f(a) + f(h) = f(2) + f(4) = 5 + 9 = 14$
> $f(a) + h = f(2) + 4 = 5 + 4 = 9$

The point of this example

- Upon looking at the original question, my gut reaction was "The question is stupid. This student is hopeless. I do not waste my time". My gut reaction was completely wrong.
- The instructor was able to involve Brett in the process and make them do some of the work.
- The instructor taught Brett that he can test his hypotheses.

2 Emily

Emily asks on piazza:

> Problem Set B asks Write a formal definition of the concept $\lim_{x\to\infty} f(x) = \infty$.
>
> My definition is as follows:
>
> Let f be a function defined on some open interval (p, ∞), where $p \in \mathbb{R}$.
> $\exists A, M \in \mathbb{R} \quad x > M \implies f(x) > A$.

Your task

- How do we help Emily?

See next page for what happened

What actually happened

Emily asks on piazza:

Problem Set B asks Write a formal definition of the concept $\lim_{x \to \infty} f(x) = \infty$.

My definition is as follows:

Let f be a function defined on some open interval (p, ∞), where $p \in \mathbb{R}$.
$\exists A, M \in \mathbb{R} \quad x > M \implies f(x) > A$.

Instructor:

Let f be the constant function 1.

Then let $A = 0.5$ and M any real number.

Then $x > M \implies f(x) > A$.

So, using your definition, I've proved that the limit of the constant function 1 is infinite.

Does that seem right?

Emily:

Not at all. Would this statement be correct then?

Let f be a function defined on some open interval (p, ∞), where $p \in R$ st

$\forall A > 0, \; \exists M > 0 \quad \text{st} \quad x > M \implies f(x) > A$

This makes more sense to me and I believe it works with the case of f being the constant function 1.

The point of this example

- The instructor did not give Emily any part of the solution, but Emily was able to fix her error and write a correct answer herself.
- The instructor taught Emily how she can test her answers.

3 Sam

Sam asks on Stack Exchange:

> Need help with this word problem, not sure how to complete this question.
>
> A cop is trying to catch drivers who speed on the highway. She finds a long stretch of the highway. She parks her car behind some bushes, 400 metres away from the highway. There is a traffic sign at the point of the road closest to her car, and there is a phone by the road 600 metres away from the traffic sign.
>
> The cop points her radar gun at a car and learns that, as the car is passing by the phone, the distance between the car and the cop is increasing at a rate of 80 km/h. The speed limit is 120 km/h. Can she fine the driver?

Helper: Have you drawn a diagram? Always start by drawing a diagram. Also, try to convert units so that they match with each other.

S: (18:25) yes, I have drawn a diagram but I don't know what to do after that.

H: (18:30) Since the police officer is facing the highway, and cars are moving on the highway, which as we can now deduce is orthogonal to the line of sight of the officer, we essentially have a triangle. Why? Because the distance between the officer and the car forms the hypotenuse of a right triangle. You also *know* what the rate of change of that hypotenuse is, 80km/h. Getting any ideas?

S: (18:32) Ok, so how would i complete the question from that point?

H: (18:33) Can you think of a relationship that relates the hypotenuse to two other sides of a triangle?

S: (18:36) No, I'm not really sure

H: (18:41) How about Pythagoras's theorem? Use that to relate the distance between the officer to the other sides of the "triangle"

S: (18:43) Could you show me the first few steps?

H: (18:50) We know that there is a triangle formed by the police officer's distance from the bushes to the highway, the distance between the police officer and the car, as well as the distance between the car to the traffic sign. Allow x to be the car's position at any time. This implies that the relationship between the police officer's distance to the car is $h^2 = (0.4)^2 + (0.6 - x)^2$. You now have a *function* relating distance to of a car, to the police officer. Note that you will have to implicitly differentiate.

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The point of this example

The student is literally learning nothing.
It is easy to get caught in this situation. It is much easier to realize what is going on when you are an outside observer, which is why reading this example is useful.
If you ever find yourself "doing the Sam example", you have my permission to stop. Doing nothing is better than "helping" this way.

The background story

We followed up on Sam because this exchange was concerning. It turns out Sam had had some problems and had not been coming to class or doing any work for two months. They just were trying to get somebody else to do their homework for them so they could submit it for marks. They dropped the course, got the help they needed with their personal situation, and re-took the course on a different term starting from scratch.

4 Li and Fan

Li asks on facebook:

> k so i'm a dumb... Find a basis and calculate the dimension of the following subspaces of R4.
> span{(2, 1, 0, -1), (-1, 1, 1, 1), (2, 7, 4, 1)}.
> I know what span, basis and dimension is, just have no clue how to do this. Anyone? Please.

Fan (another student) helps:

Fan: What are the two conditions that a set of vectors from a subspace must meet to be considered a basis?

Li: Linear independence and it spans the subspace.

Fan: Great, we know the set of vectors in the span up there span the subspace. Now do the rest

Li: but I'm not sure what to do.

Fan: What's the other condition it has to meet to be a basis

Li: There's a 3rd one? Then idk..

Fan: No... which of "Linear independence and it spans the subspace" has been met so far and which one hasn't..

Li: The linear independence.

Fan: Yeah so you need to check that for those set of vectors

Li: Okay. Let me do that and get back.

Li: Okay well, they aren't linearly independent.

Fan: What is the redundancy theorem?

Li: Or, that means that one of the vectors can be expressed by the other two vectors, so can I just get rid of one and check if two are linearly independent?

Li: Or is that the redundancy theorem?

Fan: yup

Li: And if two vectors are linearly independent then it's the basis right. And if they aren't, then I can just repeat and get rid of one vector?

...

The point of this example

Fan is a student. The facebook group was student-run, for students only, without instructor supervisor. And yet Fan was able to help Li figure things out without doing the work for them.

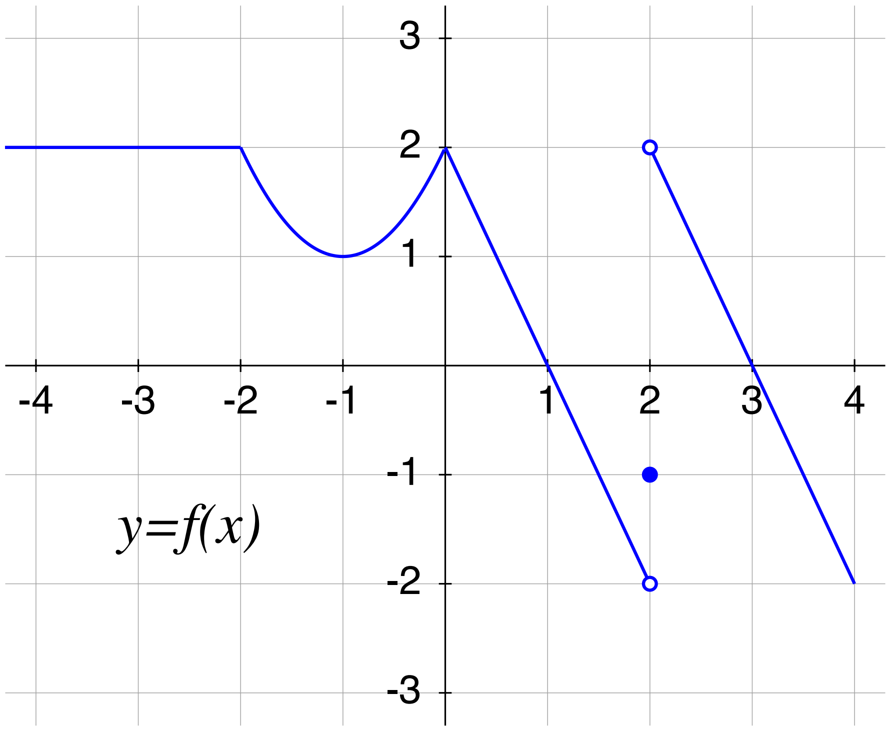
You cannot let an undergraduate student outdo you. If they can do it, so can you.

The background story

It gets even better. On the previous term Fan had been penalized for academic misconduct in a different course. A different student had asked for help on a homework assignment and Fan had posted their complete solutions. We had to penalize Fan for that. We followed with a friendly meeting to explain, not only it is wrong, but it not helpful for other students to be given answers, and what alternative ways to help there are. We left amicably. Imagine my surprise when I find Fan doing this one term later. Again: if Fan can do it, so can you.

5 A limit from a graph

Calculate $\lim_{x \to 0} f(f(x))$



Your questions

1. Guess most common answers.
2. How do we help students?

The background of this example

- This is a great learning question for a calculus class, but it is a **bad question** for an "Ask. Don't Tell" session for training of beginning TAs. It is too hard. We do not use it anymore for this specific purpose.
- I have given this question to thousands of students to work on in my classes. I have also helped dozens of students one-on-one with it during office hours. I have attempted many paths before I found one that would help students. I had to use trial and error, because my intuition was wrong.
- Sometimes "Ask. Don't Tell" is hard. Sometimes it is not clear what will work.

6 An optimization problem

Question

> We cut a one-meter piece of wire into two pieces; we make a square with one piece, and a circle with the other piece. We want to maximize the total area of the circle plus the square. What are the lengths of the pieces?

A student answer:

> 56 cm and 44 cm.

Your task

1. What error did the student make?
2. How do we help them?

What the student did

- They modelled the problem
- They found the only critical point
- They stopped. They had found the minimum, not the maximum. The maximum is at an endpoint.

The point of this example

- Use your math knowledge: the circle is the figure that maximizes the area for a given perimeter. Making the most efficient figure is going to give a higher total area than actually breaking the wire in two pieces.
- Use your teaching experience: students like to find critical points and stopping, assuming they have found the answer.
- If you invoke both your math experience and your teaching experience (who said you could get away with only one of the two?), you can figure out what the correct answer is and guess what the student probably did wrong in a mater of seconds.
- It gets better, you can tell the student

  "You found the maximum. Great. Now find the minimum as well, please".

  and then you walk away. This is a beautiful example of "Ask. Don't Tell". You are not giving anything about the solution to the student, and yet is enough for the student (with enough time) to realize that something is wrong, what it is, and hopefully to solve the problem.
- Sometimes we have TAs who are skeptical of this whole activity. This is the example that will win them over.

7 A test answer

(From the blog http://learningcurves.blogspot.ca)

The question on the test

> Without factoring it, explain how the number $N = (1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10 \cdot 11) + 1$ can be used to argue that there is a prime larger than 11.

The student's real answer

> Because $(1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10 \cdot 11)$ equals a positive number because the number is multiplied by 2. However when one number is added it creates a negative number that is greater than 11, therefore there is a larger prime number than 11.

Your task

- The student's answer looks nonsensical, but I promise it made perfect sense in their mind. Once you figure it out, you will know you are right. What were they thinking?

Solution

The student is using "positive" and "negative" to mean "even" and "odd". Then they are using the "fact" that all odd numbers are prime.

The point of this example

Understanding what the student is doing or the error they made is sometimes difficult. However, if we want to use "Ask. Don't Tell", we need to.